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THESIS

A STUDY OF CONNECTED REPLENISHMENT AT SEA BASED ON LOAD REQUIREMENTS THROUGH DATA ANALYSIS AND COMPUTER SIMULATION

by

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ABSTRACT

By analyzing CONREP data for oiler (AO) and ammunition (AE) ships replenishing destroyers (DD) and attack carriers (CVA) with regard to day and night operation and load requirement, it is shown through Chi-Square goodness-of-fit tests that alongside time data can be fitted to an Erlang or exponential distribution. In addition, several methods for estimating the shape, scale, and shift parameters for gamma and Erlang distributions are presented.

Employing well known but analytically little used features of the replenishment operation, a computer simulation model is then formulated and programmed based on these distributions, and thus sensitive to changes in load requirements. Using the simulation as an experimental device, an example is run to demonstrate its use in estimating the time to complete the operation and conducting sensitivity analyses on the various load requirements.

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I. INTRODUCTION

The primary reason the United States Navy is capable of performing its regularly assigned mission as well as assist in global contingencies effecting United States' interests, is the ability to swiftly resupply combatant ships almost anywhere in the world. This is possible through replenishment at sea operations which render ships independent of costly, vulnerable land based supply facilities while effectively increasing combatant force strength, since ships are able to remain on or near their assigned stations during replenishment.

Although very important, replenishment at sea is a hazardous operation. During replenishment, participant ships are extremely close increasing their possibility of collision and vulnerability to enemy attack. Even though vertical replenishment (VERTREP) using helicopters has helped to reduce the risk in some cases, all fuel transfers such as Navy Special Fuel Oil (NFSO), Aviation Gasoline (AVGAS), and JP-5, and most transfers of dry material such as ammunition and stores, are necessarily done by the more dangerous connected replenishment (CONREP) method. Recognizing the importance of CONREP, the U.S. Navy is using a considerable amount of its resources to increase the efficiency and reduce the danger of this operation. Increased training of personnel, redesign or replenishment ships and equipment, and studies utilizing analytical and computer simulation models are efforts that have been and are being made in this endeavor.

Operations Research studies of the replenishment at sea operation have been very few. However, McCoullough [4] formulated two models of a simplified underway replenishment operation. One an analytical

infinite cycle model using multi-stage queueing theory and the other a non-cyclic computer simulation. In both models the distribution of service time at each replenishment ship was assumed to be exponential. Waggoner [7] and to some extent Patterson [6] also used the assumption of exponential service (i.e. alongside) times in replenishment at sea operations. Besecker [1] questioned the validity of this assumption and analyzing CONREP data, showed that the distribution of service times can be fitted with a gamma distribution. Categorizing the data as to participants and day and night operation, he contended that the data would also fit the more useful Erlang distribution in each case. This paper continues the investigation of replenishment at sea operations along the lines of reference [1].

II. OBJECTIVES

The most immediate gain in CONREP efficiency can be experienced through proper planning of the operation. However, studies of the operation have been largely ignored in the planning stage, since they lack the realism necessary to gain the confidence of operational commanders. Previous models (such as [4] and [7]) have not taken full advantage of the many known features of CONREP, for instance, arrivals at supply ships are not random but may be promulgated in advance by the Task Group Commander. Also, the quantity required by each combatant ship is known prior to the operation to some degree of accuracy rather than being a random variable. If the transit time between various supply ships is considered deterministic and known, the uncertainty in the system is a result only of the unknown alongside times, which should be dependent on the participants and the loads transferred.

The first objective of this paper is to develop a more accurate estimation of alongside times for various combinations of CONREP participants by considering a given load requirement. Such conditional distributions would be applicable in analytical models, in computer simulations, and possibly in planning replenishment at sea operations, with more accurate predictions resulting in each case.

The formulation and programming of a simulation model based on these alongside time distributions, and thus sensitive to changes in load requirements is the second objective. While incorporating the above known features, the simulation should be designed to enable its

user to easily adjust the arrangement of the ships involved, load requirements, and distribution parameters. In this regard, he could utilize it to test the suitability of various arrangements of combatant ships and the sensitivity of load changes.

III. SOURCE OF DATA

Data used to obtain the distribution of alongside completion times were from replenishment at sea (RAS) reports 3180/1A and 3180/1B for the period December 1967 to April 1969. Commander Service Force, U.S. Pacific Fleet requires these reports be made to him immediately following underway replenishment by those replenishment ships under his cognizance. Each report includes the replenishment ship type and hull number, receiving ship type and hull number, total alongside time, quantity transferred in barrels or short tones as appropriate, UNREP speed, whether day or night, whether EASTPAC or WESTPAC, and whether or not there was simultaneous port and starboard UNREP in progress. Full instructions concerning the submission of these reports are available in COMSERVPAC INST 3180.3F of 6 November 1967.

Alongside time is defined as the elapsed time from commencing approach from 300 yards astern to time of departure. Only data submitted by oiler (AO) and ammunition (AE) type ships involving destroyer (DD) and attack carrier (CVA) recipients were used, since a sufficient quantity was available only for these combinations. However, sufficient data should become available in subsequent years to make similar analyses possible for all combinations of participants.

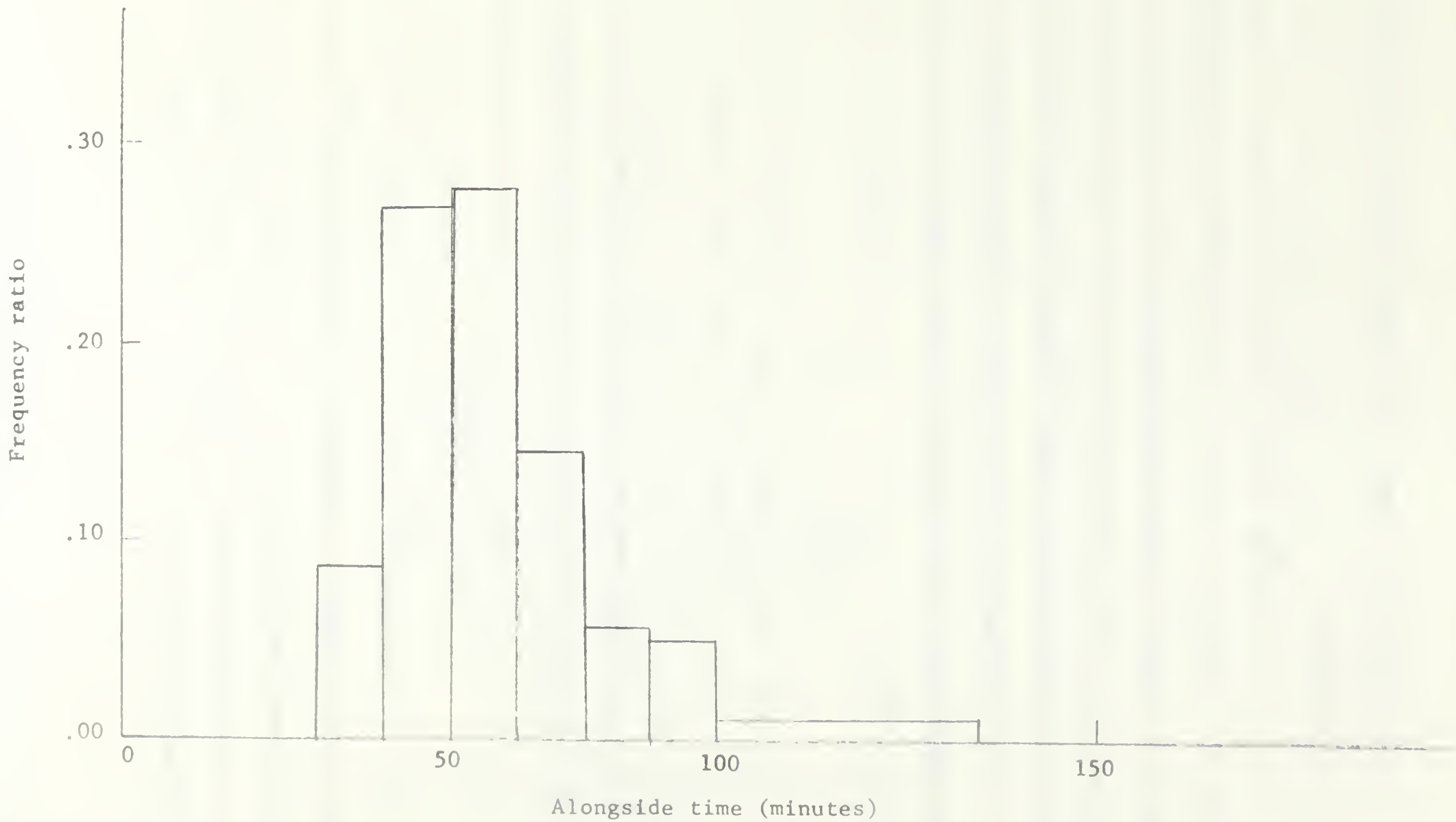
IV. METHOD OF DATA ANALYSIS

In order to accomplish the objective of a more accurate estimation of alongside time distributions for various participants, data was first categorized as to type of ships involved, then separated with regard to day or night operation, and finally subdivided into sets based on quantity transferred. The distinction based on day or night conditions was considered necessary, since from personal experience and as shown by Besecker, night operations consistently take longer than those conducted in daylight hours. The load intervals were selected on the basis of quantity of data available and uniformity of interval ranges. The latter was desirable in order to establish load intervals necessary for the computer simulation.

Data in each category was analyzed by constructing histograms of the relative frequency with which the alongside replenishment time fell within ten minute intervals. The last interval is always a half open interval from its selected lower bound to infinity. In each case the histogram suggested that the distribution of the completion times for the respective load intervals could be fitted to a gamma or exponential distribution presented in Appendix A. This was expected since the distribution of alongside times over all intervals had been fitted to a gamma distribution in [1].

Figure (1) is the histogram constructed for the case in which an oiler (AO) was transferring 1000-1099 barrels of fuel to a destroyer (DD) during daylight hours. This is presented as a typical example.

Figure 1. Histogram of the relative frequency with which the alongside replenishment time fell within ten minute intervals for AO vs. DD - day, 1000-1099 barrels.



It was appropriate to apply a Chi-Square goodness-of-fit test to the null hypothesis

$$H_o : P_i = P_{io} \quad i - (1,\ldots,k)$$

against all alternatives, where P_i is the probability of finding a completion in the i^{th} interval with respect to the distribution in question, P_{io} is the relative frequency of the data falling within the same interval, and k is the number of intervals. The probability P_i was calculated using Besecker's computer subroutine GAMDIS with the gamma parameters estimated from the sample mean and variance using the relationships

$$\hat{r} = \frac{\bar{T}^2}{s^2}$$

$$\hat{\lambda} = \frac{\bar{T}}{s^2}$$

where $\bar{T}$ is the sample mean and s^2 the sample variance.

The Chi-Square test consisted of calculating the statistic

$$Q = \sum_{i=1}^{K} \frac{(X_i - nP_{io})^2}{nP_{io}}$$

where X_i is the number of datum which fell within the i^{th} ten minute interval and n is the sample size. In accordance with statistical theory 3 , if H_o is true, then the random variable Q has an approximate Chi-Square distribution. The hypothesis was accepted if

$$Q < \chi^2\ (.95,\ k-3).$$

The degrees of freedom was necessarily k-3 since two parameters were estimated from the data. Many authors caution that nP_i, $i = 1,2, \ldots , k$

should be greater than five, since Q has only an approximate Chi-Square distribution. This warning was strickly adhered to, which necessitated combining two or more time intervals when the quantity of data in a load interval was small.

In the cases involving attack carriers, the hypothesis was accepted for all load intervals. Since the Erlang distribution is more ameanable to analytical and computer simulation models and only differs from the gamma distribution in that $\hat{r}$ is restricted to be a positive integer, each $\hat{r}$ was rounded to the nearest positive integer, the parameter $\hat{\lambda}$ adjusted according to the expression

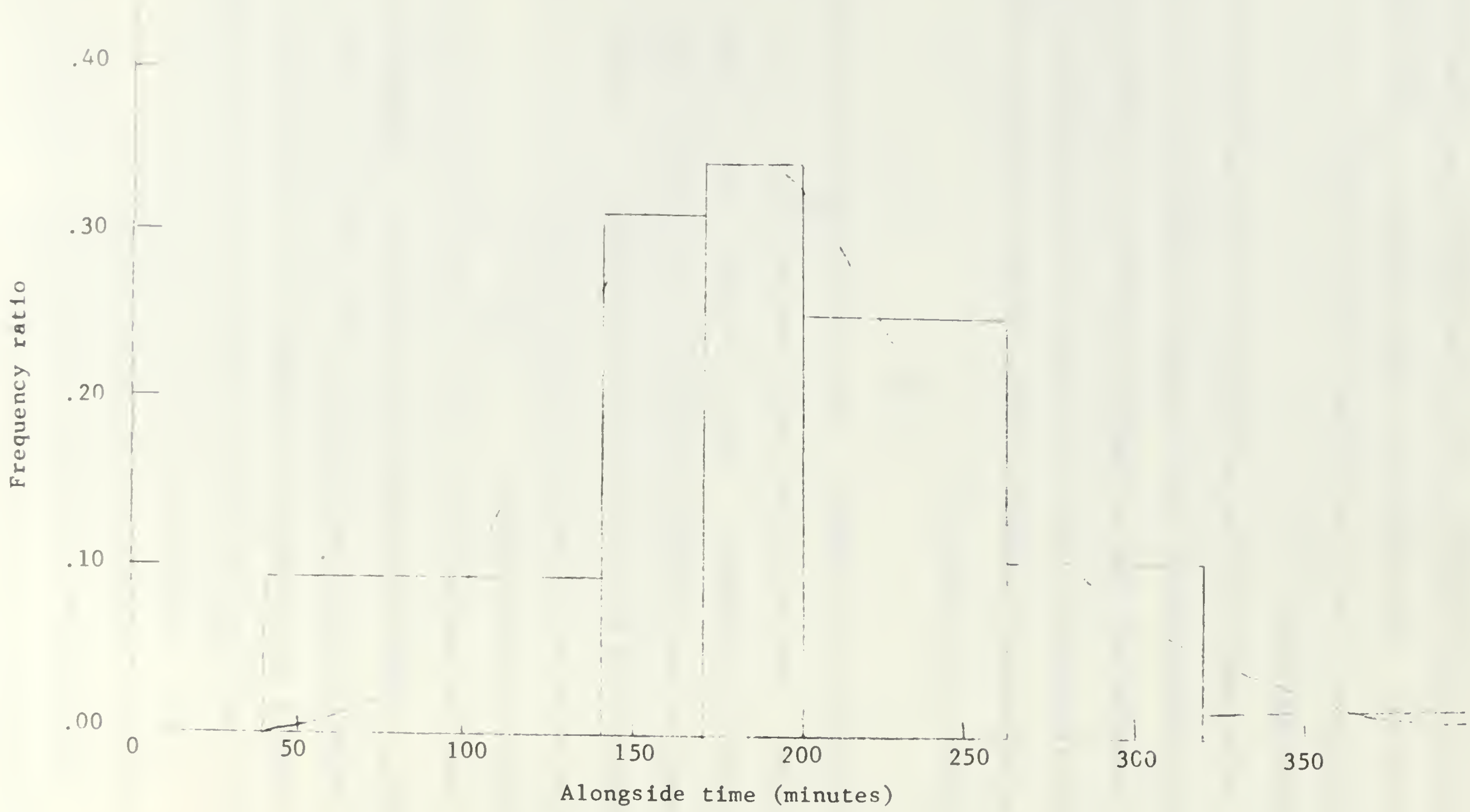
$$\hat{\lambda} = \frac{\hat{r}}{\bar{T}}$$

and a similar test conducted. Again, the hypothesis was accepted in each case. Results of these latter tests are given in Tables I, II, III, and IV in Appendix B. Figure (2) shows a typical example with the probability density function superimposed on the relative frequency histogram for a case involving AO-CVA participants. The figure illustrates the method of combining intervals consistent with the greater than five rule given above.

When the above method was used for cases involving destroyer type ships, the hypothesis was rejected in each case. Re-examination of the histograms for these cases suggested that a shift of the time axis by an amount a would lend to an acceptance of the gamma distribution. Several approaches were taken to estimate the distribution parameters r, , and a. Although many were not successful in this study, all are presented for reference in future data analyses.

Noting that the first three centeral moments of the gamma distribution given by

Figure 2. Probability density function suggested by the method of moments superimposed on the respective relative frequency histogram for a typical AO vs. CVA case.



$$E\left[T\right] = \frac{r}{\lambda} + a, \tag{1}$$

$$E\ (T - \mu)^2 = \frac{r}{\lambda^2},$$

and

$$E\ (T - \mu)^3 = 2\frac{r}{\lambda^3}$$

could be used to estimate the parameters, the following relations were derived:

$$\hat{r} = 4\frac{s^6}{m_3^2}, \tag{2}$$

$$\hat{\lambda} = 2\frac{s^2}{m_3} \tag{3}$$

$$\hat{a} = \bar{T} - 2\frac{s^4}{m_3} \tag{4}$$

where $\bar{T}$ is the sample mean given by

$$\bar{T} = \frac{1}{n}\sum_{i=1}^{n} T_i,$$

s^2 is the sample variance given by

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(T_i - \bar{T})^2,$$

and m_3 is the third centeral moment given by

$$m_3 = \frac{1}{n}\sum_{i=1}^{n}(T_i - \bar{T})^3.$$

By rounding the shape parameter r to the nearest positive integer and adjusting the scale and shift parameters, the following expressions were obtained:

$$\hat{r}_o = \left(4 \frac{s_6}{m_3^2}\right)^*,$$

$$\hat{\lambda} = \frac{\sqrt{\hat{r}_o}}{s^2},$$

and

$$\hat{a} = \bar{T} - s\sqrt{\hat{r}_o}$$

where $(x)^*$ is the nearest integer to x. Using this method in several cases, $\hat{r}_o$ was found to be unity in each case suggesting an exponential distribution. Figure (3) shows a typical case with the exponential distribution superimposed on the histogram. The hypothesis was rejected for these cases as clearly suggested by the figure.

Concern over the poor results and the fact that observed times were less than $\hat{a}$, prompted another investigation into the method of obtaining the shift parameter. Hypothesizing an exponential distribution and constructing the likelihood function

$$L(\lambda, a) = \begin{cases} \lambda^n e^{-\lambda n\bar{T} + n\lambda a} & \text{for } a \le \min\limits_{1 \le i \le m} T_i \\ 0 & \end{cases}$$

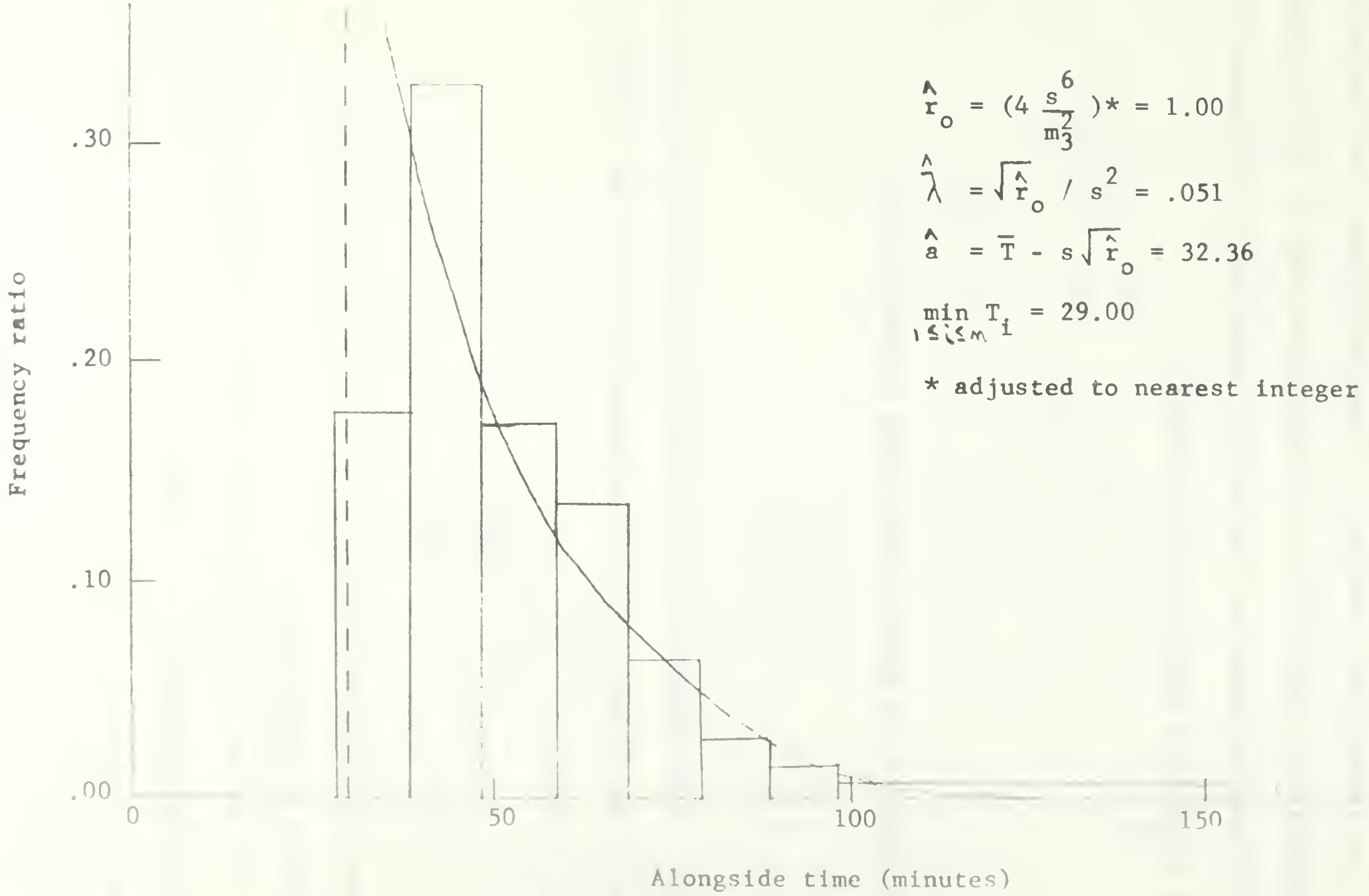
the maximum likelihood estimator for a was obtained as

$$\hat{a} = \min_{1 \le i \le m} T_i \ .$$

This implied that

$$\hat{\lambda} = \frac{1}{\bar{T} - \hat{a}}$$

Figure 3. Suggested exponential distribution superimposed on the respective relative frequency histogram for a typical AO vs. DD case.



from Eq. (1). However, for all cases examined using this method, the hypothesis was rejected on the basis of the Chi-Square goodness-of-fit test as illustrated in Figure (4).

Concluding that rounding to obtain an Erlang distribution was inappropriate when $0 \le \hat{r} \le 1$, an attempt was made to use the maximum likelihood estimate for a under the assumption of the gamma distribution. The likelihood function in this case is

$$L(a) = \begin{cases} \lambda^{\bar{T}n} \prod_{i=1}^{n} (T_i - a)^{r-1} e^{-\lambda \sum_{i=1}^{n} (T_i - a)} & \text{for } a \le \min_{1 \le i \le n} T_i \\ 0 & \text{otherwise,} \end{cases}$$

resulting in the maximum likelihood estimate of a as

$$\hat{a} = \min_{1 \le i \le n} T_i ,$$

again. Then referring to the expressions obtained using the method of moments and adjusting the other parameters to a, the following equations were derived:

$$\hat{r} = \frac{(\bar{T}-a)^2}{s^2}$$

$$\hat{\lambda} = \frac{\bar{T}-a}{s^2} .$$

Gamma distributions suggested by this third method did not fit the data as shown in Figure (5).

In another attempt to use the method of moments to estimate the distribution parameters, no adjustment was made to the estimates given by Eqs. (2), (3), and (4). But this also proved unsatisfactory as illustrated in Figure (6).

Convinced that the gamma distribution was appropriate from the shape of the histograms based on ten minute intervals from zero,

Figure 4. Suggested exponential probability density function superimposed on the respective relative frequency histogram for a typical AO vs. DD case with a estimated using the maximum likelihood function.

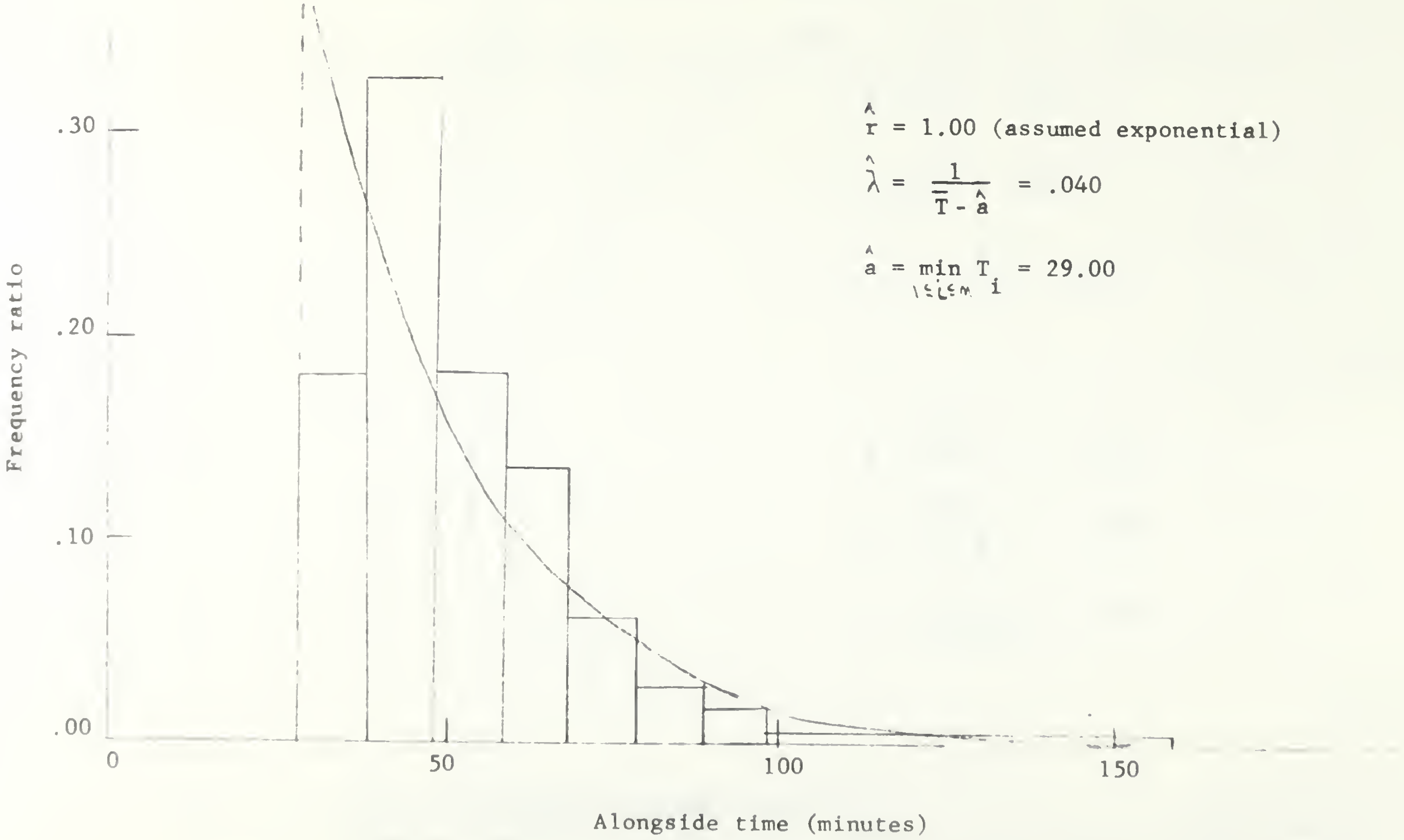
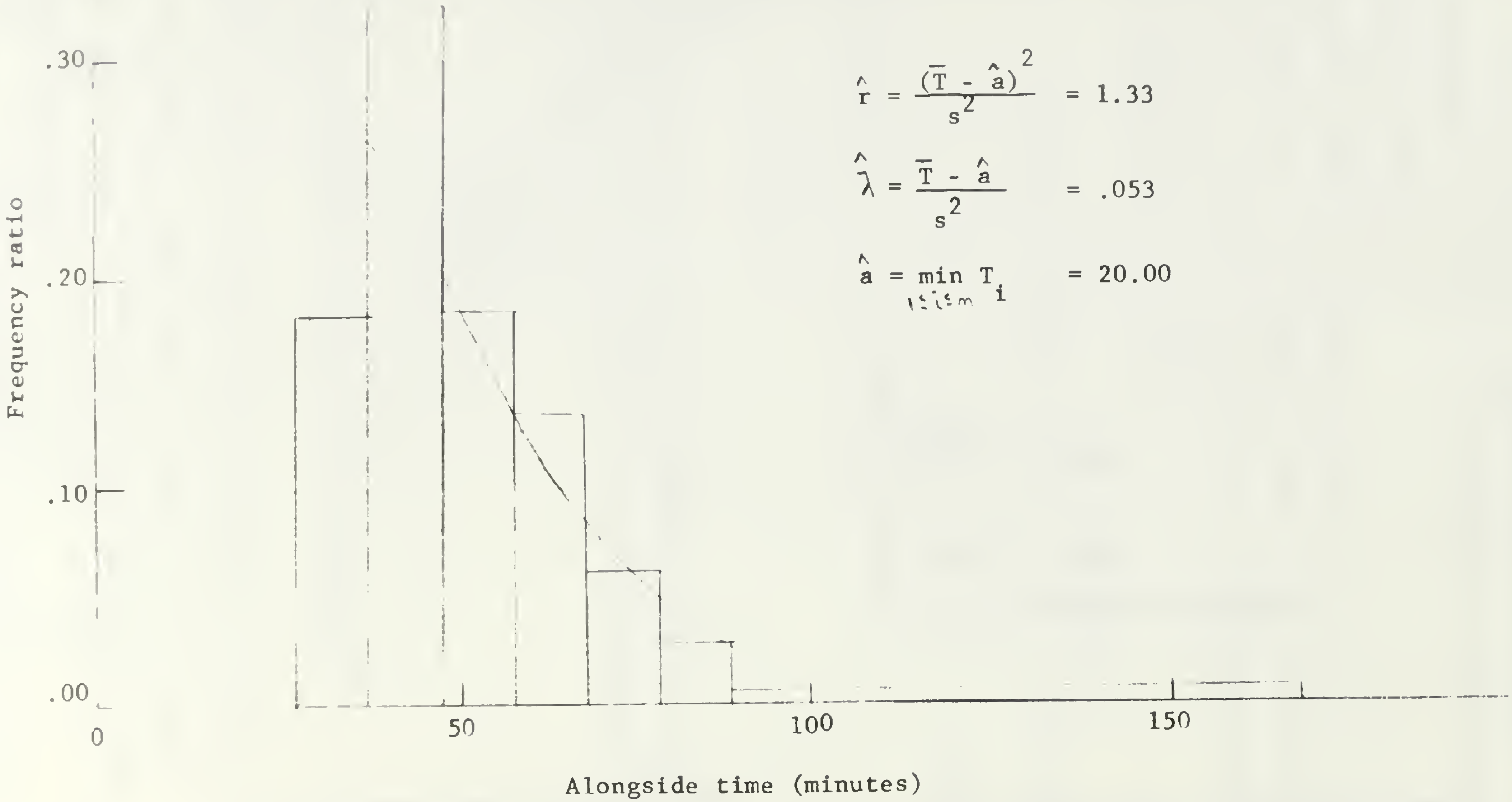


Figure 5. Suggested gamma probability density function superimposed on the respective relative frequency histogram for a typical AO vs. DD case with a estimated from the maximum likelihood function.

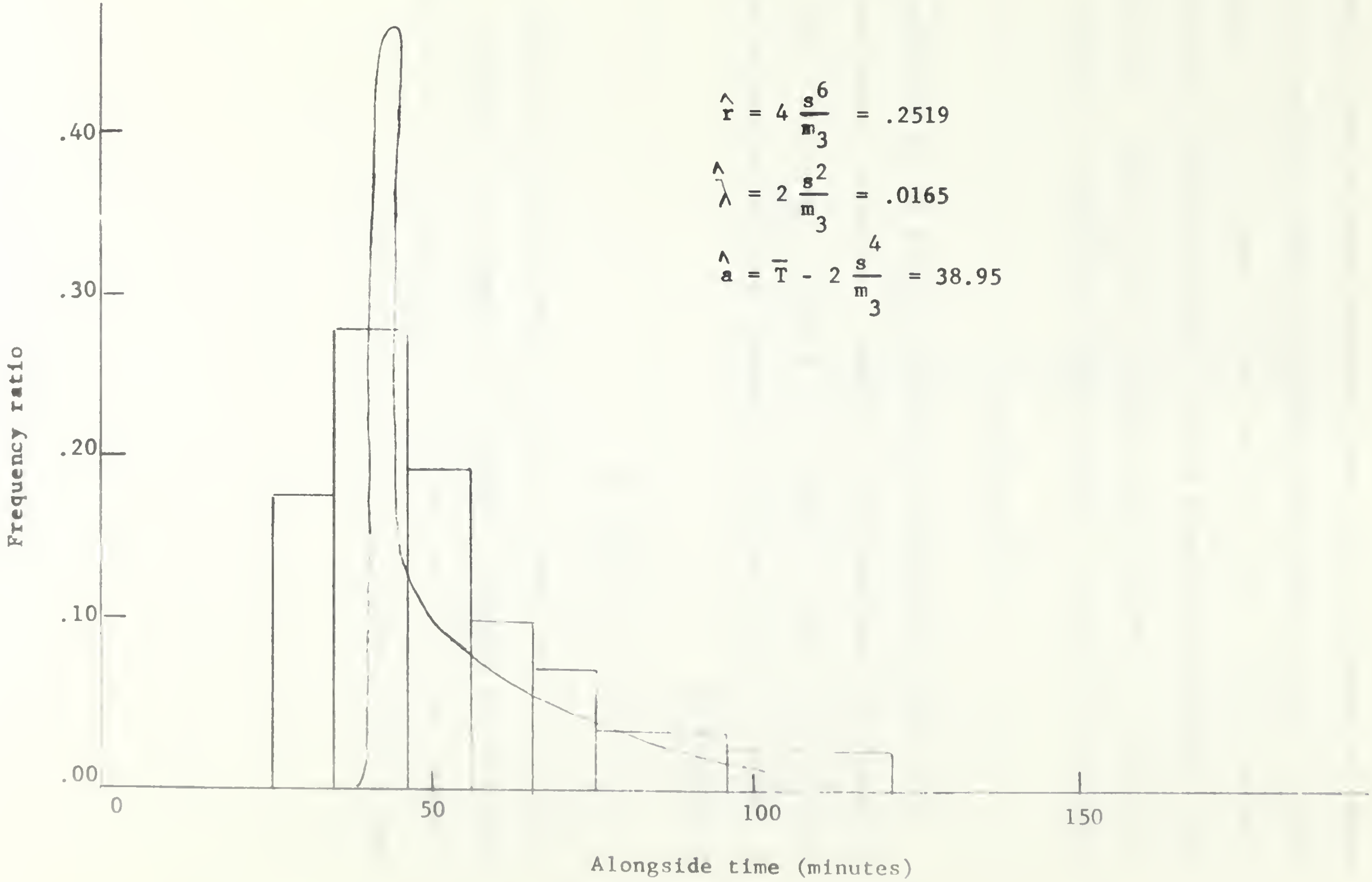


$$\hat{r} = \frac{(\bar{T} - \hat{a})^2}{s^2} = 1.33$$

$$\hat{\lambda} = \frac{\bar{T} - \hat{a}}{s^2} = .053$$

$$\hat{a} = \min_{1 \le i \le m} T_i = 20.00$$

Figure 6. Suggested gamma probability density function superimposed on the respective relative frequency histogram for a typical AO vs. DD case with all parameter estimates unadjusted.



extensive probability tables were generated using Besecker's subordinate GAMDIS. Then by scanning these tables for similar patterns of the relative frequency ratios of time intervals compared to the probability that a completion time would be within the same interval, an Erlang distribution was selected that appeared to fit the data. A Chi-Square goodness-of-fit test was conducted and the hypothesis H_o accepted if

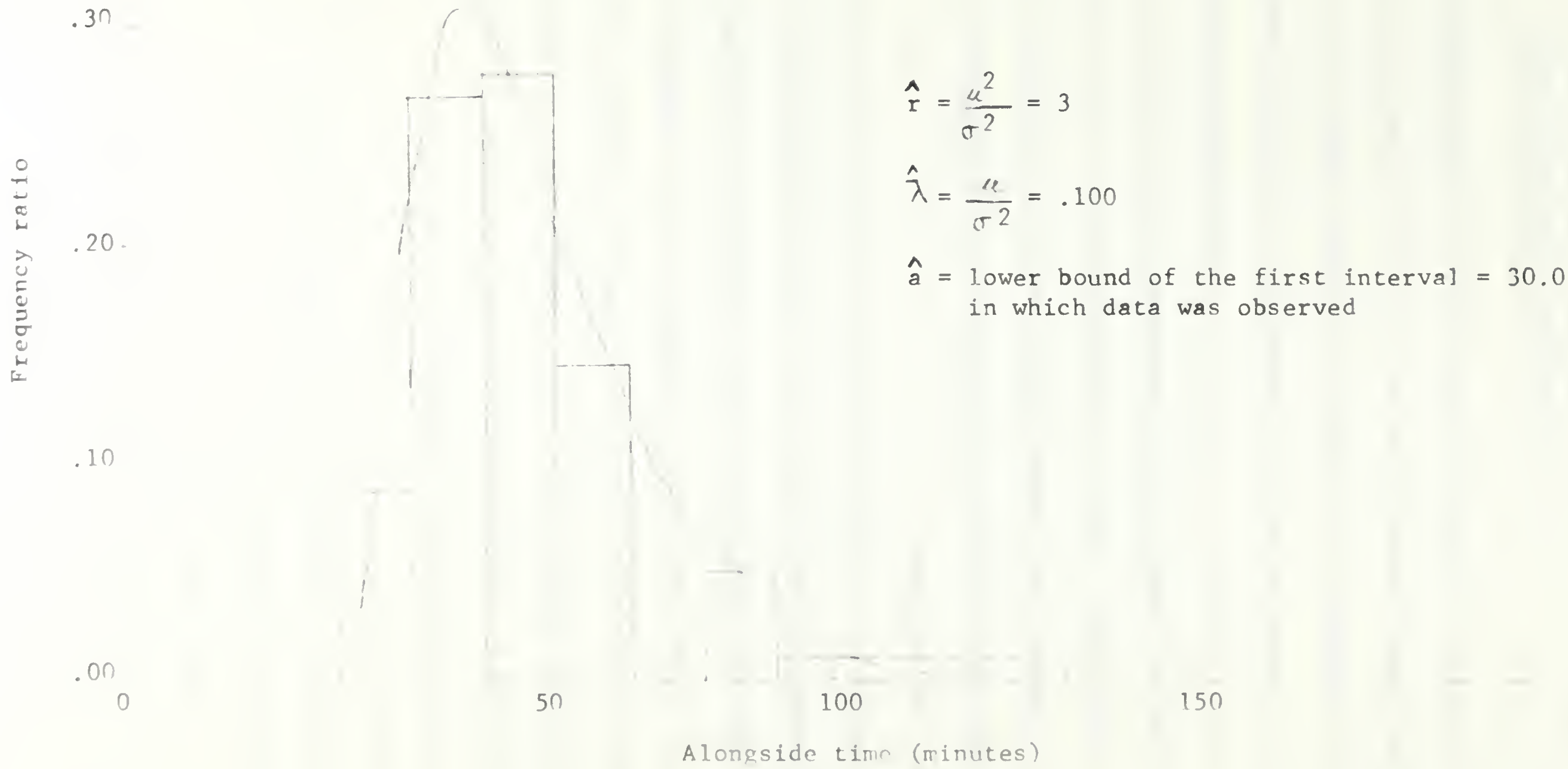
$$Q \quad \chi^2 (.95,k-1).$$

This method proved quite fruitful, resulting in the acceptance of the hypothesis associated with an Erlang distribution in each case. A typical case is shown in Figure (7). Results of these tests are given in Tables V, VI, VII, and VIII in Appendix B with T calculated using the expression

$$\hat{T} = \frac{\hat{r}}{\hat{\lambda}} + \hat{a}.$$

The estimate a was chosen as the lower bound of the first interval in which data was observed. Figures (12) through (56) in Appendix C illustrate the fit of the probability density functions and the histograms for each case involving destroyer type ships.

Figure 7. Suggested gamma probability density function using scanning method imposed on the respective relative frequency histogram for a typical AO vs. DD case.



V. THE COMPUTER SIMULATION MODEL

As stated in Naylor [5] the purpose of simulating a large operation such as replenishment at sea is to conduct situational experiments that would ordinarily be too expensive and/or cumbersome to perform physically. Because of the many conceivable combinations of participant ships and variations in quantities transferred between them that one might want to examine, physical experimentation would be unreasonable here. Therefore, based on the distributions of the previous section, a Monte Carlo type digital computer simulation of the replenishment at sea operation named SIMA was devised and programmed in accordance with the second objective.

SIMA was designed to accommodate a system of from one to four replenishment ships, composing an Underway Replenishment Group (URG), with each ship having one or two replenishment stations on the port or port and starboard sides. A maximum of twenty combatant ships can be in the queue at each station. Furthermore, six types of combatant ships can be in the system. This capacity is well beyond what one would normally expect during an actual replenishment operation, therefore overload of the model is not expected.

Drawing from personal experience, the model was formulated as realistically as possible, thereby taking advantage of the known factors of a replenishment at sea system. All combatant ships are initially distributed among the various stations of the supply ships, with each individually characterized by the load it requires from each replenishment ship. Combatant ships proceed sequentially through the system and upon arrival at a replenishment ship it enters the smaller

(if more than one exists) queue. Each ship receives the services of every supply ship and, having done so, is ejected from the system. Although the combatant ships do pass through the system in a specified manner from their initial positions, these conditions closely resemble the ability of a Task Group Commander to formulate the operation prior to its commencement, utilizing his knowledge of the approximate loads transferred among the participants.

Using the simulation as an experimental device, one is able to specify the participants, number of stations, initial sizes of the station queues, and quantities transferred between the respective ships, and then periodically observe the state of the system. These observations are made at variable time increments which are the random alongside completion times generated as per Appendix D. An observation is made and the system advances whenever any one of the URG ships has completed a replenishment. Each observation consists of the time since the start of the operation and the state of the system. The state of the system at these times is the number of ships, including the one being served, at each replenishment station and is represented by an n dimensional vector, where n is the number of stations in the system. The i^{th} component of the state vector being the number of ships at the i^{th} station.

This model was programmed in Fortran IV and tested on the IBM model 360-67 digital computer at the U.S. Naval Postgraduate School, Monterey, California. A printout of the program is presented in Appendix E. Because of the digital nature of the simulation, ship types were given the following identification numbers:

Oiler - 1
Ammunition ship - 2
Destroyer - 1
Attack carrier - 2

Since all the parameters derived in the previous section are given in data statements, they may be revised by changing the statements. In using this technique, a user need only supply the following information to conduct replenishment at sea experiments:

1. Type and sequence of supply ships
2. Day or night operation
3. Type, initial position, and load requirement of the combatant ships.

This information is provided by a data deck. The first m(m - 1,2,3, or 4) cards represent the m supply ships, each card having the following format:

Columns 1-3	Type of replenishment ship, i.e. AO, AE
Columns 4-5	Number of stations available. (1 or 2)
Columns 6-7	Supply ship identification number
Columns 8-10	Blank
Column 11	Must be either 0,1, or blank. 0 or blank implies that there is at least one other replenishment ship in the system and another card must be read. 1 implies no more supply ships are in the system and the program continues to the next statement.
Columns 12-80	Blank

The order in which these cards appear designates the sequence in which combatant ships pass through the system. The m+1st card specifies day or night operation and has the following format:

Column 1	Blank
Column 2	Must be either 1 or 2. 1 designates day 2 designates night
Columns 3-80	Blank

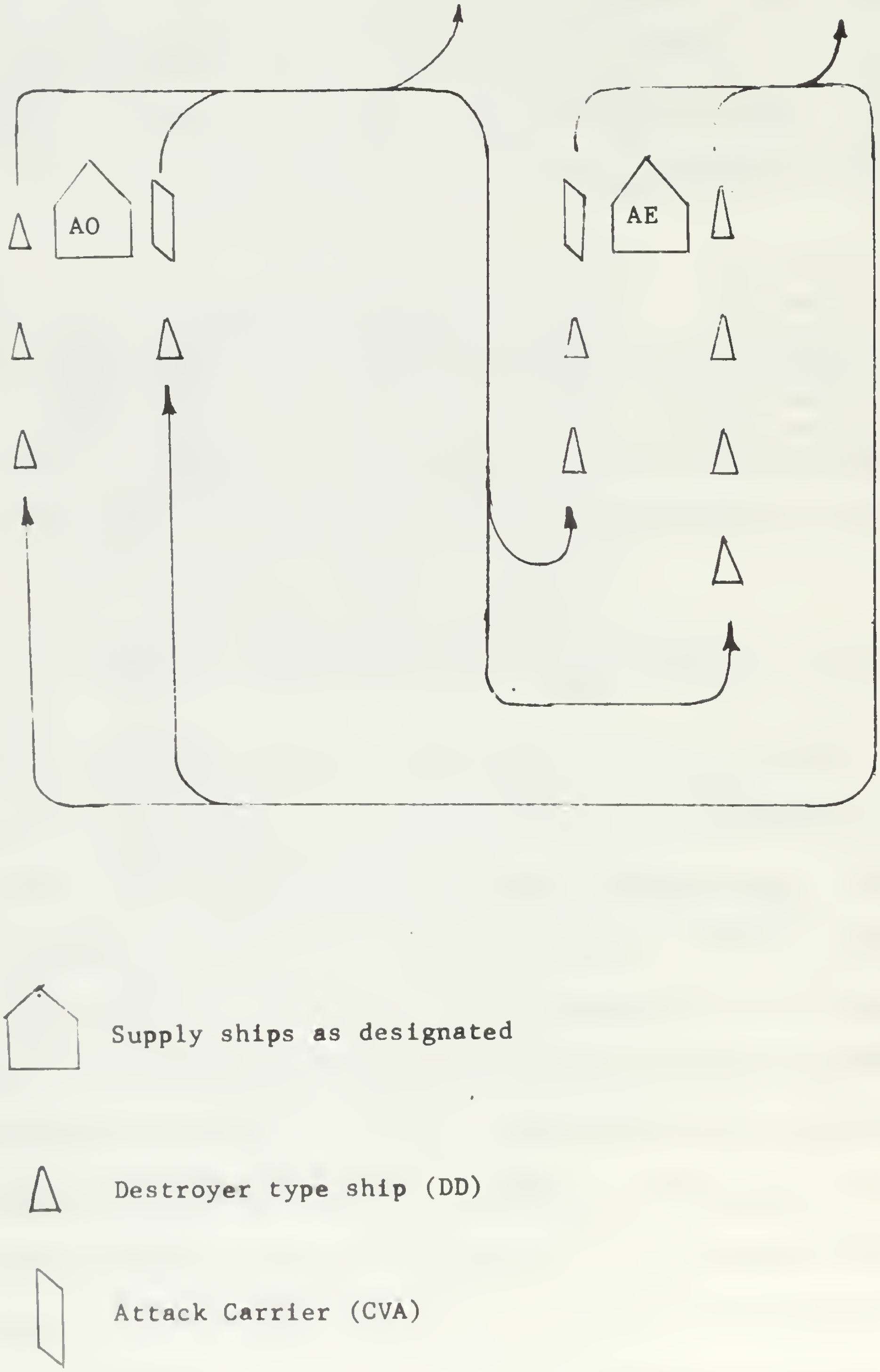
The remaining cards give information concerning the combatant ships, with the cards arranged as these ships initially appear in the system.

The remaining cards give information concerning the combatant ships, with the cards arranged as these ships initially appear in the system. The initial condition of each station's queue for each supply ship is given in accordance with the order of the supply ship cards, with the port station designated first. Each card represents a combatant ship with information given in the following format:

Columns 1-3	Ship type, i.e. DD, CVA
Columns 4-5	Ship identification number.
Columns 6-11	Load required from first supply ship.
Columns 12-17	Load required from second supply ship.
Columns 18-23	Load required from third supply ship.
Columns 24-29	Load required from fourth supply ship.
Column 30	Must be either 0, 1, or blank. 0 or blank indicates that there is at least one other combatant ship in the queue for the station in question and another card must be read. 1 indicates that no more ships are initially in the queue in question.
Columns 31-80	Blank

Figure (8) shows a hypothetical initial arrangement of the supply and combatant ships, illustrating the possible routes of the combatant ships. As an example, this system was simulated using SIMA in day and night operation. Figure (9) shows the data cards submitted to set up the system for the daylight case. Since only two supply ships are in the example, the results may be represented graphically as in Figures (10) and (11). The origin represents the starting time of the operation. Each dot represents a combatant ship having completed replenishment at either the AO or AE. The length of the line segment between two consecutive dots indicates the elapsed time in minutes between two consecutive completions of the replenishment operation. For example, in Figure (10) a ship departed the AO at approximately 60 minutes, a ship left the AE at approximately 80 minutes, a ship left the AO at

Figure 8. Hypothetical initial arrangement of supply and combatant ships.



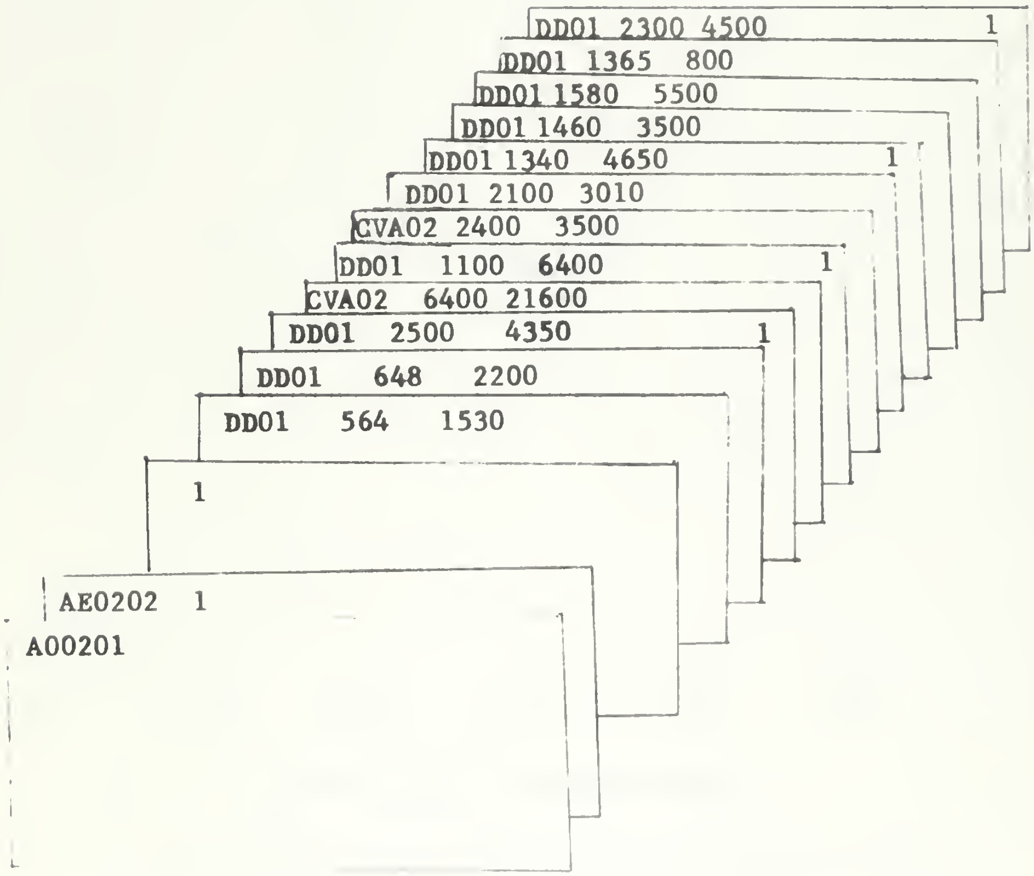


Figure 9. Data deck used to set up example problem.

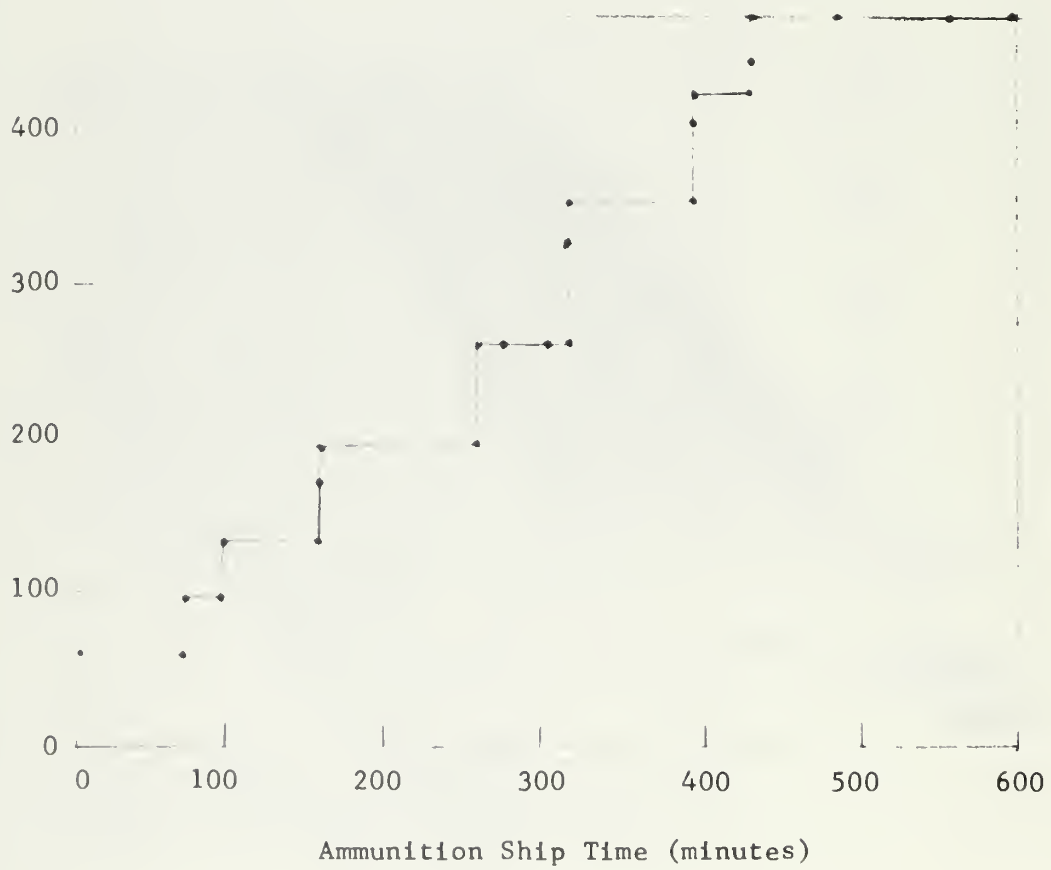


Figure 10. Graphical representation of simulation results for the example system during daylight operation.

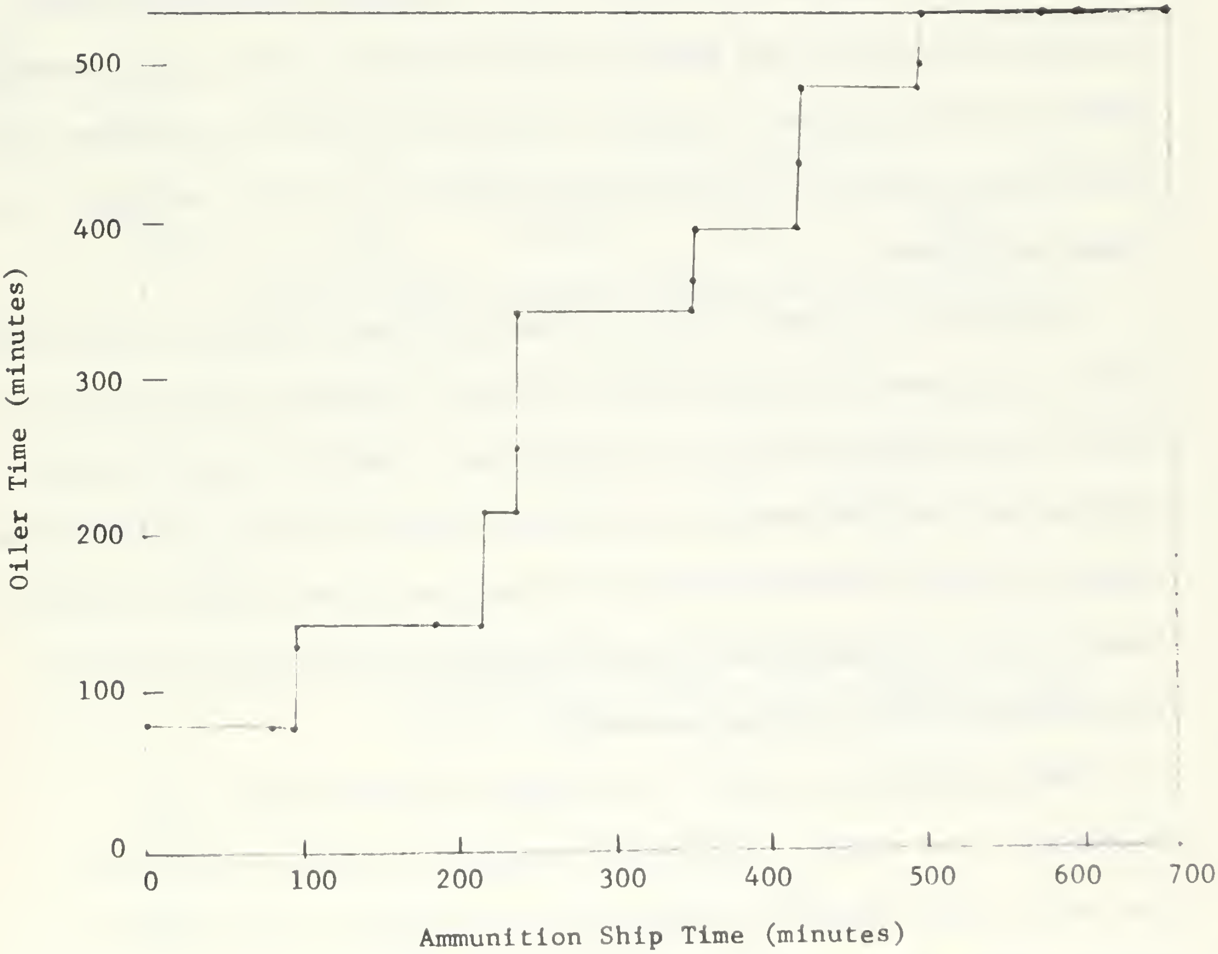


Figure 11. Graphical representation of simulation results for the example system during night operation.

approximately 100 minutes, a ship left the AE at approximately 100 minutes, and so on, regarding the commencement of the replenishment operation 0 hours and 0 minutes. Knowing the initial arrangement of the ships, one is able to reconstruct the movement of each combatant ship through the system and its departure time from the supply ships.

Upon running the simulation fifty times for the example, it was found that the expected (average) time to complete the replenishment operation during daylight hours was 598.62 minutes while night operations required 635.38 minutes. Figures (12) and (13) are the histograms for the frequency ratios of total replenishment time of all combatants for these experiments.

We see from the data cards that the destroyer, initially the second ship in the queue at the port side of the AO, requires 648 barrels of fuel oil and 2200 short tons of ammunition. When this was changed to 1100 barrels and 3000 short tons and the simulation rerun, the expected time to complete the operation was 605.54 for day and 638.10 for night conditions. Sensitivity analyses along these lines could easily be accommodated by the simulation model.

SIMA might also be useful in planning replenishment at sea operations. By tracing the path of particular ships in simulation, one could detect possible bottlenecks in a proposed replenishment system. Then by repositioning one or more ships in the formation and conducting additional experiments, one could plan a more efficient arrangement to be used in actual replenishment operations.

Figure 12. Relative frequency histogram for total replenishment time of all combatants from fifty simulation runs for day operation.

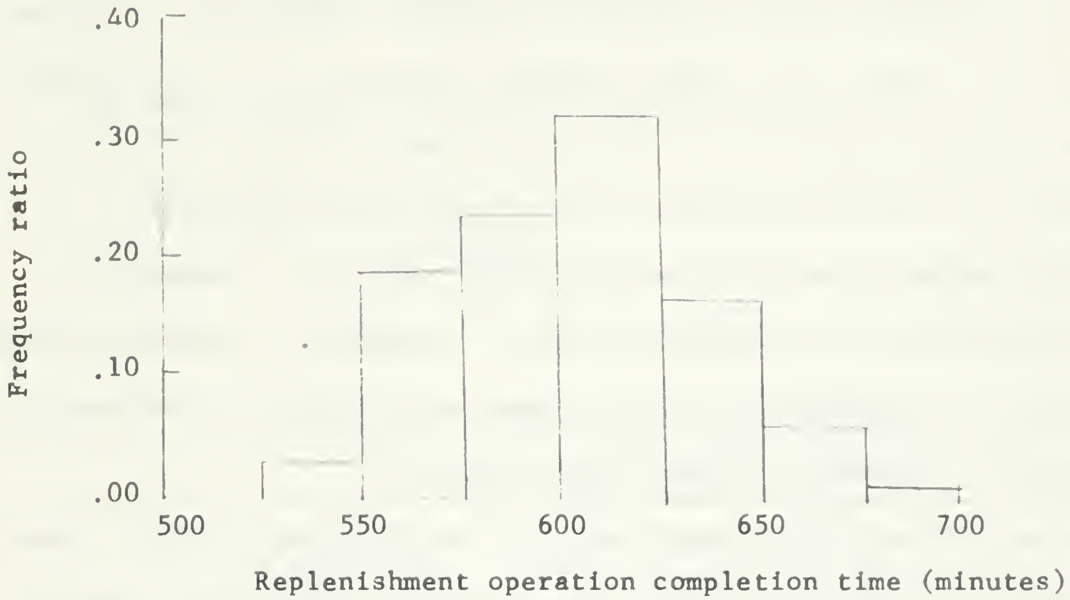
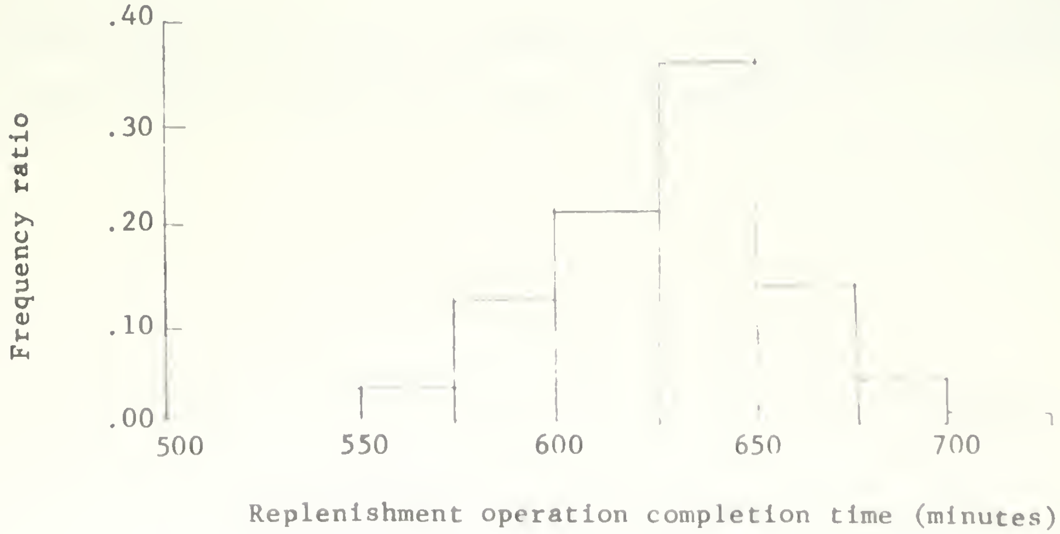


Figure 13. Relative frequency histogram for total replenishment time of all combatants from fifty simulation runs for night operation.



VI. SUMMARY

In this paper we have shown that an Erlang or exponential distribution may be fitted to the CONREP alongside time data involving AO, AE, DD, and CVA type ships for each of several load intervals. It is possible for these distributions to be used in analytical models employing Laplace transforms that would be sensitive to changes in load requirements of the combatant ships. Furthermore, several methods for obtaining the parameters for the gamma and Erlang distributions are presented for reference in future data analyses.

The second part of the study utilizes the distributions in a computer simulation model. Using the simulation as an experimental device, one can easily and realistically investigate the suitability of various CONREP situations confident that the results will reflect one's knowledge of the approximate loads required by each of the combatant ships. It was demonstrated that SIMA could be used to estimate the time to complete the operation and investigate the sensitivity of certain load requirements, and suggested that the model be used in planning underway replenishment formations.

APPENDIX A

The Gamma, Erlang, and Exponential Distributions

The gamma probability density function is given by

$$f(x) = \begin{cases} \dfrac{\lambda^r x^{r-1} e^{-x}}{\Gamma(n)} & \text{for } x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

where r is the shape parameter and λ the scale parameter. In shifting the function by an amount a the random variable x would be replaced by x-a in the above expression.

The Erlang and exponential distributions are special cases of the gamma distribution. If r is a positive integer, the probability density function may be written as

$$f(x) = \begin{cases} \dfrac{\lambda^r x^{n-1} e^{-x}}{(n-1)!} & \text{for } x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

and is that of the Erlang distribution. If r is unity, the distribution is the well known exponential distribution.

APPENDIX B

Tables

Table I. AO vs. CVA - day

load int.	$\bar{T}$	$\hat{\lambda}$	$\hat{r}$	Q	$\chi^2_{(.95,k-3)}$
0-3999	85.0	.096	8	2.615	5.99
4000-4999	98.7	.244	24	2.716	3.84
5000-5999	115.9	.117	14	3.444	5.99
6000-6999	127.2	.074	9	1.497	3.84
7000-7999	142.2	.050	7	4.651	5.99
8000-8999	139.3	.156	22	2.411	3.84
9000-9999	126.3	.047	6	1.218	5.99
10000-	143.3	.060	4	2.584	5.99

Table II. AO vs. CVA - night

load int.	$\bar{T}$	$\hat{\lambda}$	$\hat{r}$	Q	$\chi^2_{(.95,k-3)}$
0-5999	118.2	.068	8	1.007	5.99
6000-7999	134.1	.088	12	1.154	5.99
8000-9999	139.9	.063	9	0.856	5.99
10000-11999	141.8	.136	19	0.732	5.99
12000-	194.2	.056	11	0.635	5.99

Table III. AE vs. CVA - day

load int.	$\bar{T}$	$\hat{\lambda}$	$\hat{r}$	Q	$\chi^2_{(.95,k-3)}$
0-9999	74.7	.092	7	2.486	3.84
10000-19999	105.2	.114	12	2.624	3.84
20000-29999	159.9	.111	18	3.055	5.99
30000-39999	189.8	.141	27	2.180	3.84
40000-	229.4	.079	18	0.067	3.84

Table IV. AE vs. CVA - night

load int.	$\bar{T}$	$\hat{\lambda}$	$\hat{r}$	Q	$\chi^2_{(.95,k-3)}$
0-19999	90.1	.084	8	2.573	3.84
20000-29999	162.8	.033	6	0.541	3.84
30000-39999	168.8	.076	13	1.402	3.84
40000-49999	216.7	.113	24	0.649	3.84
50000-	258.1	.094	24	0.823	5.99

Table V. AO vs. DD - day

load int.	$\bar{T}$	$\hat{T}$	$\hat{\lambda}$	$\hat{r}$	$\hat{a}$	Q	$\chi^2_{(.95,k-1)}$
0-399	41.3	40.0	.100	2	20	4.558	11.10
400-499	44.9	50.0	.100	3	20	2.385	9.49
500-599	49.8	50.0	.098	2	30	1.973	9.49
600-699	43.1	40.0	.100	2	20	1.104	9.49
700-799	47.7	51.4	.140	3	30	3.172	7.81
800-899	52.9	53.3	.060	2	20	7.912	12.60
900-999	52.8	55.0	.120	3	30	7.143	9.49
1000-1099	49.6	55.0	.120	3	30	4.413	14.10
1100-1199	53.0	50.0	.100	3	20	5.529	12.60
1200-1299	53.4	50.0	.100	3	20	8.167	12.60
1300-1399	56.4	63.3	.140	5	30	1.389	7.81
1400-1499	54.1	55.0	.040	1	30	7.137	12.60
1500-1599	62.6	58.5	.070	2	30	4.637	12.60
1600-1699	65.3	65.7	.140	5	30	4.189	12.60
1700-1799	61.3	60.0	.100	3	30	3.306	9.94
1800-1899	64.1	60.0	.100	3	30	1.509	11.10
1900-1999	67.3	70.0	.100	3	40	2.439	9.94
2000-2099	67.4	40.0	.100	3	40	3.073	11.10
2100-2199	65.2	65.0	.040	1	40	2.306	9.49
2200-	68.3	70.0	.100	3	40	4.925	11.10

Table VI. AO vs. DD - night

load int.	$\bar{T}$	$\hat{T}$	$\hat{\lambda}$	$\hat{r}$	$\hat{a}$	Q	$\chi^2_{(.95,k-1)}$
0-599	65.6	67.5	.080	3	30	5.626	11.10
600-699	63.3	67.5	.080	3	30	1.510	9.49
700-799	70.2	67.5	.080	3	30	0.253	7.81
800-899	78.6	67.5	.080	3	30	1.784	7.81
900-999	67.8	67.5	.080	3	30	0.318	7.81
1000-1099	61.3	67.5	.080	3	30	1.313	9.49
1100-1199	62.1	67.5	.080	3	30	2.912	7.81
1200-1299	56.1	65.0	.080	2	40	1.799	11.10
1300-1399	67.6	65.0	.080	2	40	8.014	9.49
1400-1499	71.6	67.5	.080	3	30	1.528	7.81
1500-1599	55.9	65.0	.160	4	40	3.191	7.81
1600-1699	74.4	75.5	.080	3	40	0.549	7.81
1700-1799	78.8	75.5	.080	3	40	1.854	7.81
1800-	76.7	75.5	.080	3	40	4.563	9.49

Table VII. AE vs. DD - day

load int.	$\bar{T}$	$\hat{T}$	$\hat{\lambda}$	$\hat{r}$	$\hat{a}$	Q	$\chi^2(.95,k-1)$
0-999	42.8	46.0	.250	9	10	3.584	9.49
1000-1999	63.3	60.0	.100	4	20	1.013	9.49
2000-2999	77.0	80.0	.080	4	30	3.332	12.60
3000-3999	97.8	106.6	.060	4	40	7.927	14.10
4000-4999	110.5	105.4	.044	2	60	1.524	11.10
5000-	127.5	120.0	.080	4	70	5.642	12.60

Table VIII. AE vs. DD - night

load int.	$\bar{T}$	$\hat{T}$	$\hat{\lambda}$	$\hat{r}$	$\hat{a}$	Q	$\chi^2(.95,k-1)$
0-999	45.6	40.0	.100	3	10	5.634	9.49
1000-1999	72.3	67.5	.080	3	30	1.075	9.49
2000-2999	90.9	87.5	.080	3	50	2.913	9.49
3000-3999	104.7	100.0	.040	2	50	3.666	9.49
4000-	139.7	145.0	.080	3	70	3.149	12.60

APPENDIX C

Figures

This appendix contains figures illustrating the fit of the probability density functions to the histograms for all cases involving destroyer type ships.

Figure 14. AO vs. DD - day. 0-399 barrels

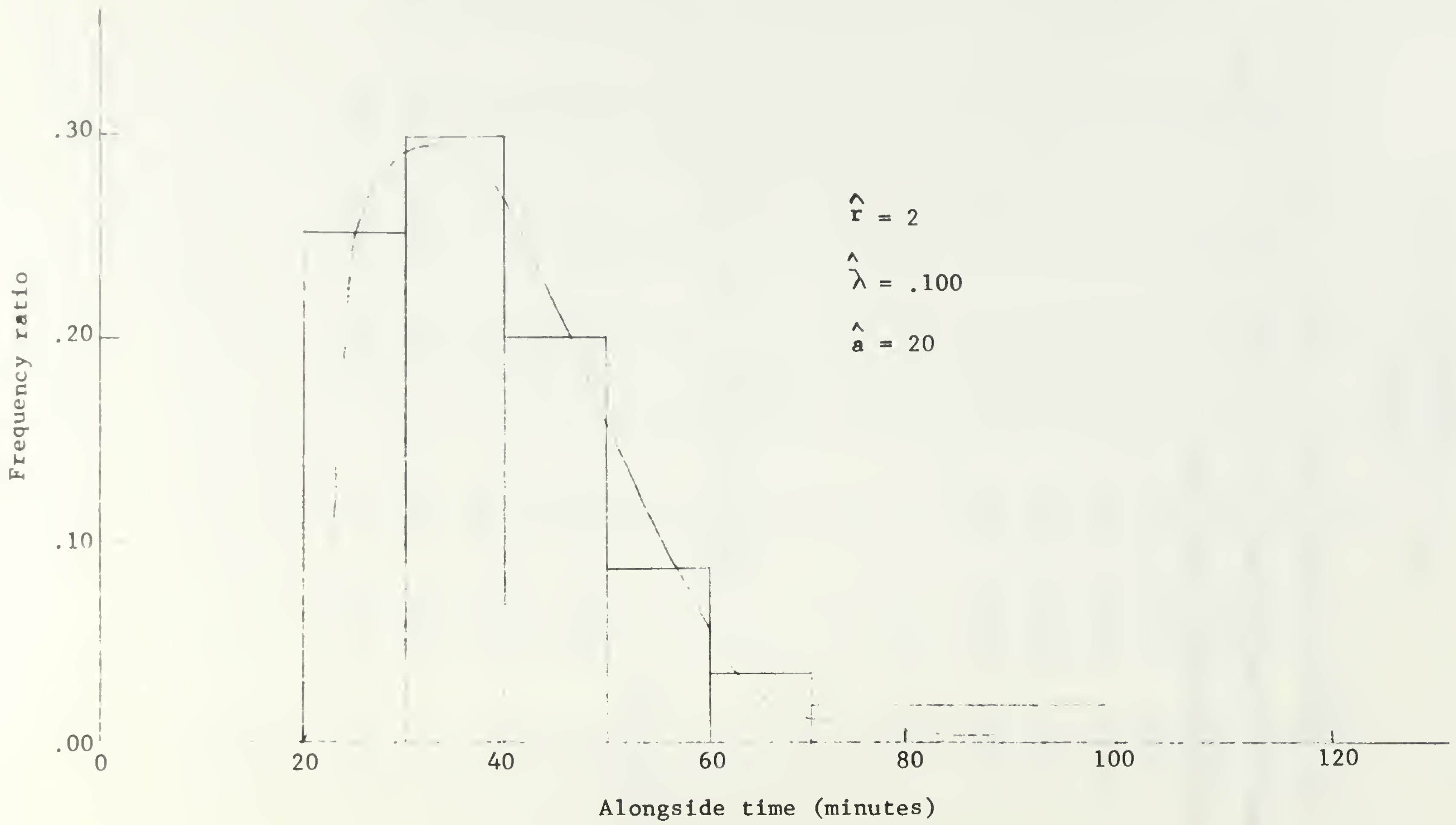


Figure 15. AO vs. DD - day. 400-499 barrels

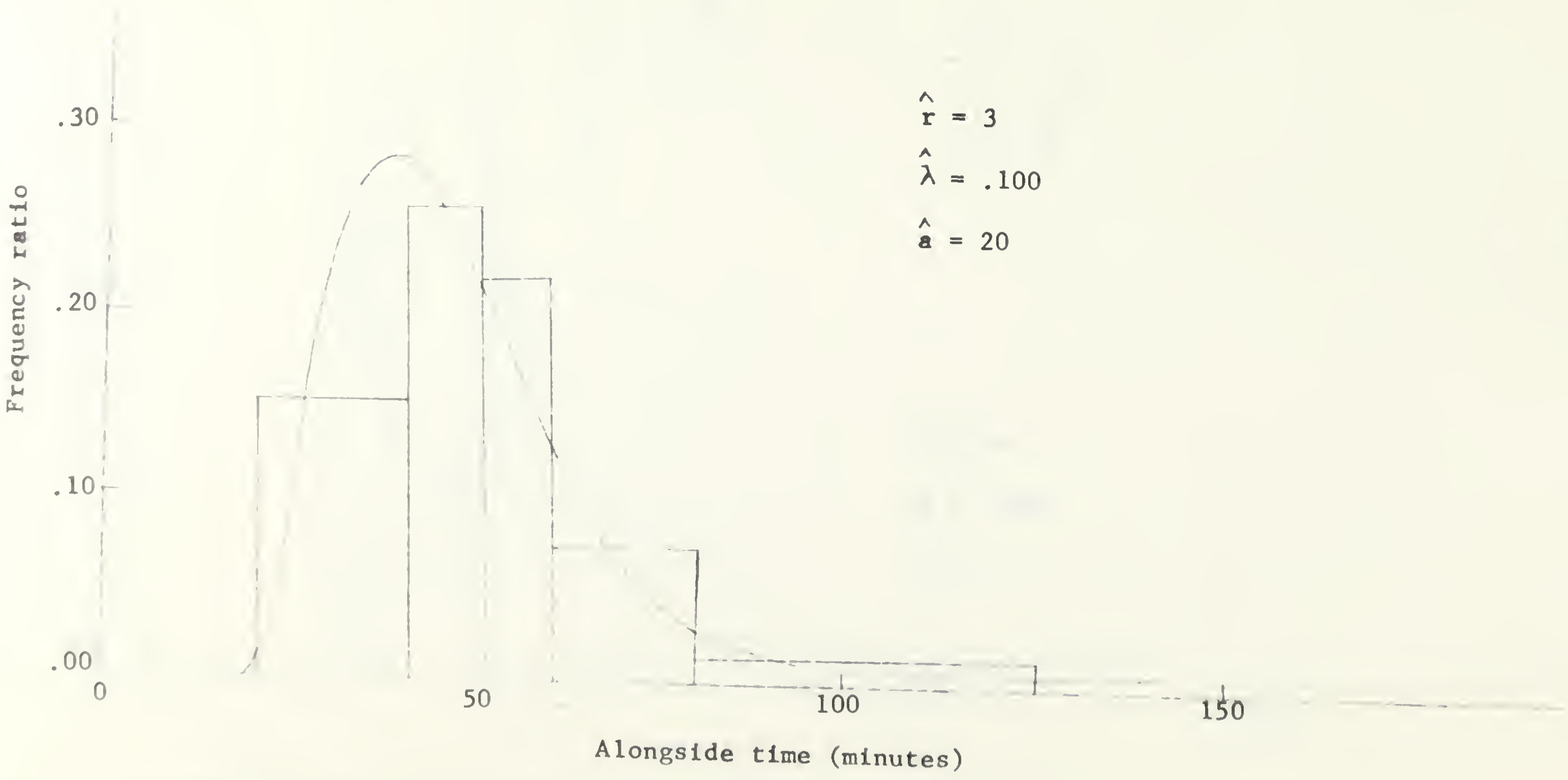


Figure 16. AO vs. DD - day. 500-599 barrels

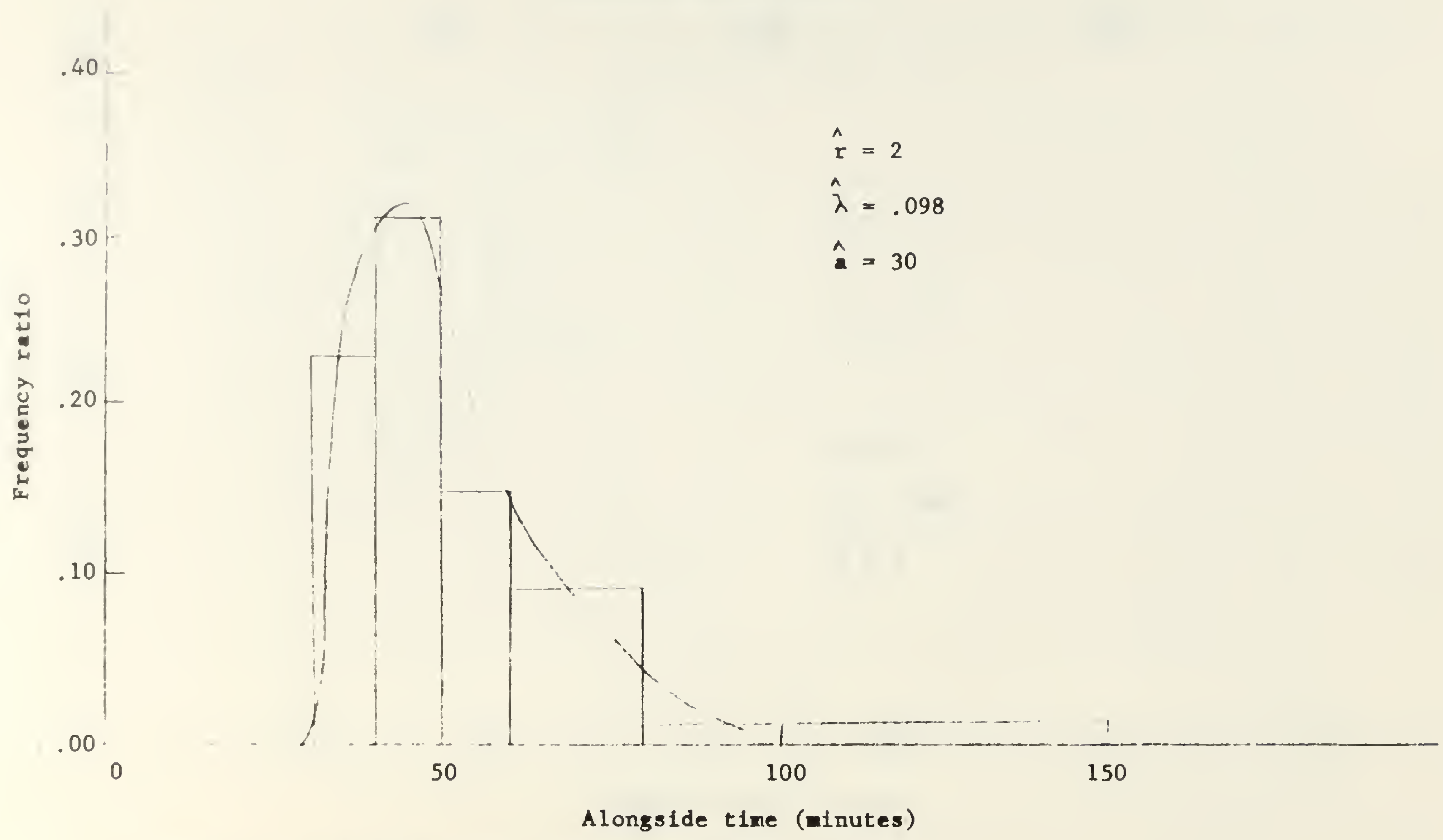


Figure 17. AO vs. DD - day. 600-699 barrels

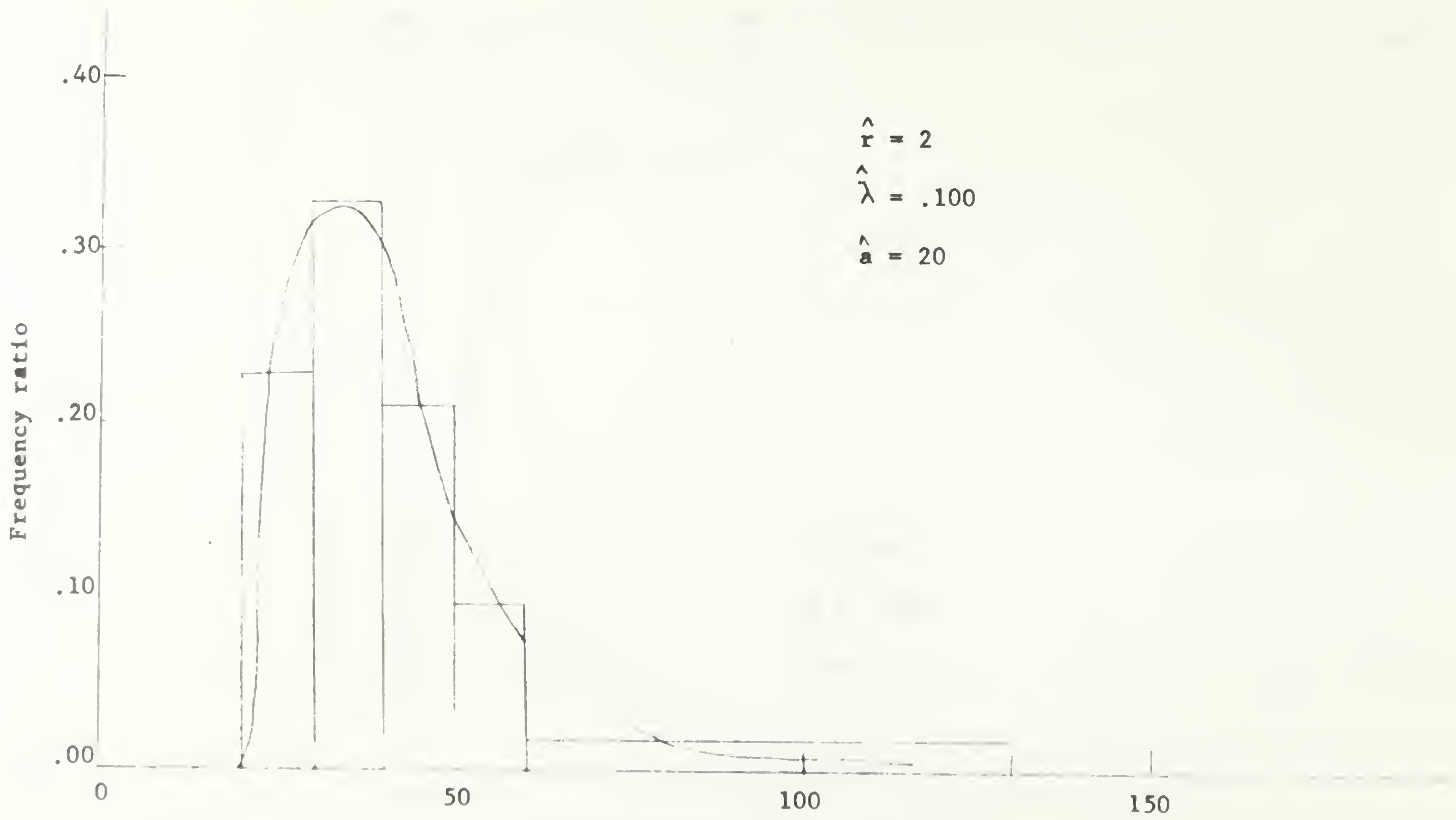


Figure 18. AO vs DD - day. 700-799 barrels

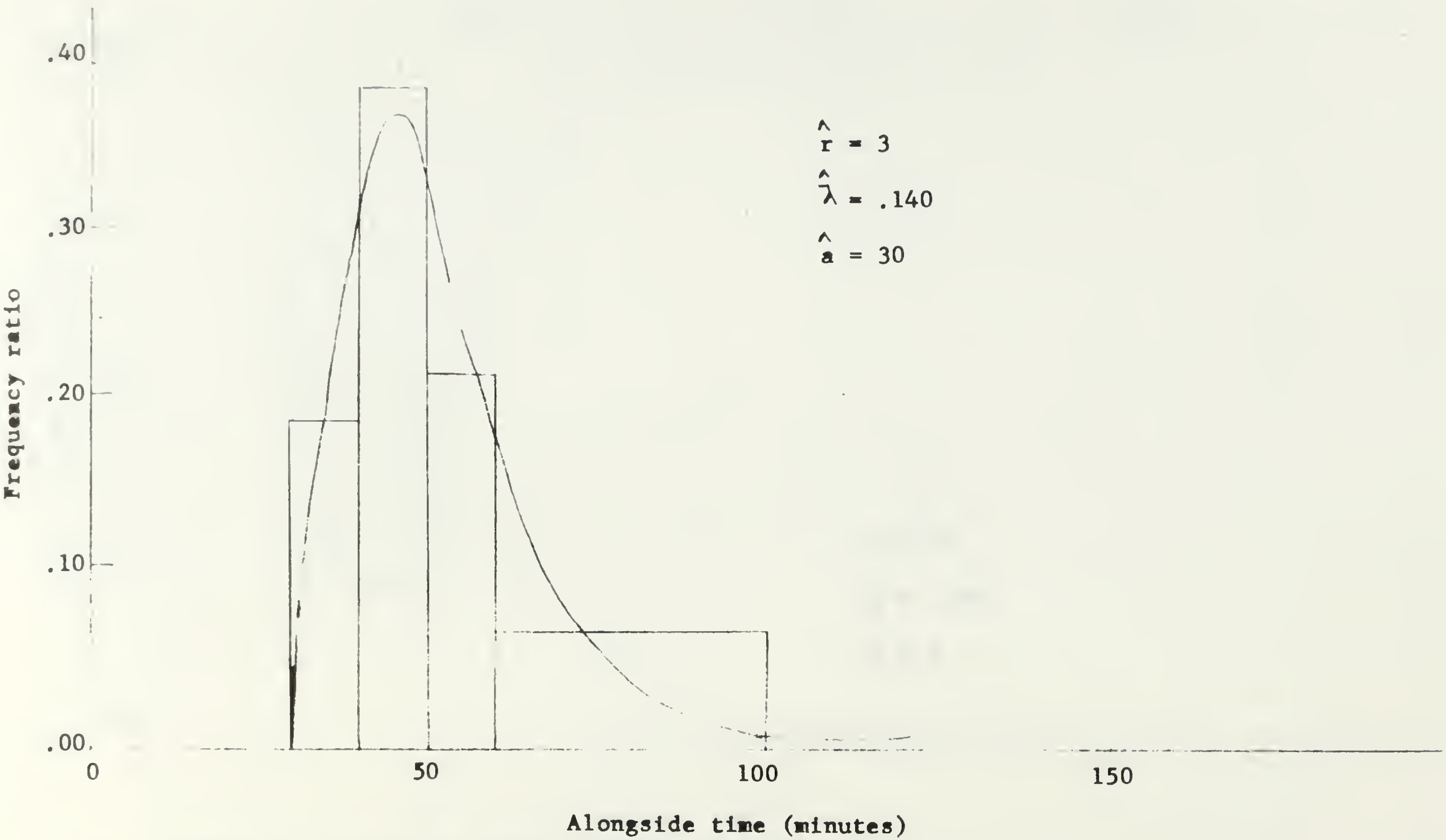


Figure 19. AO vs DD - day. 800-899 barrels

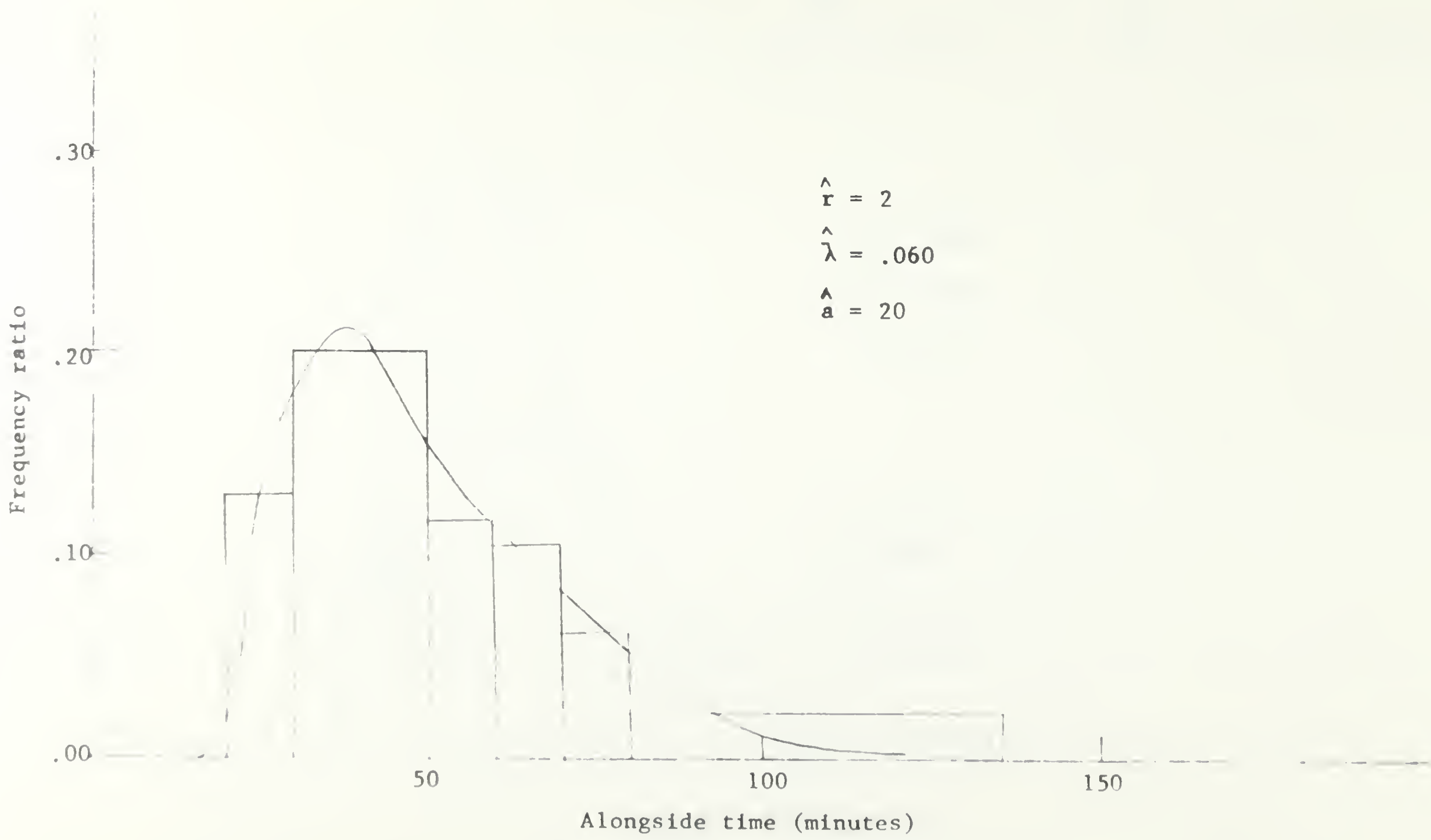


Figure 20. AO vs. DD - day. 900-999 barrels

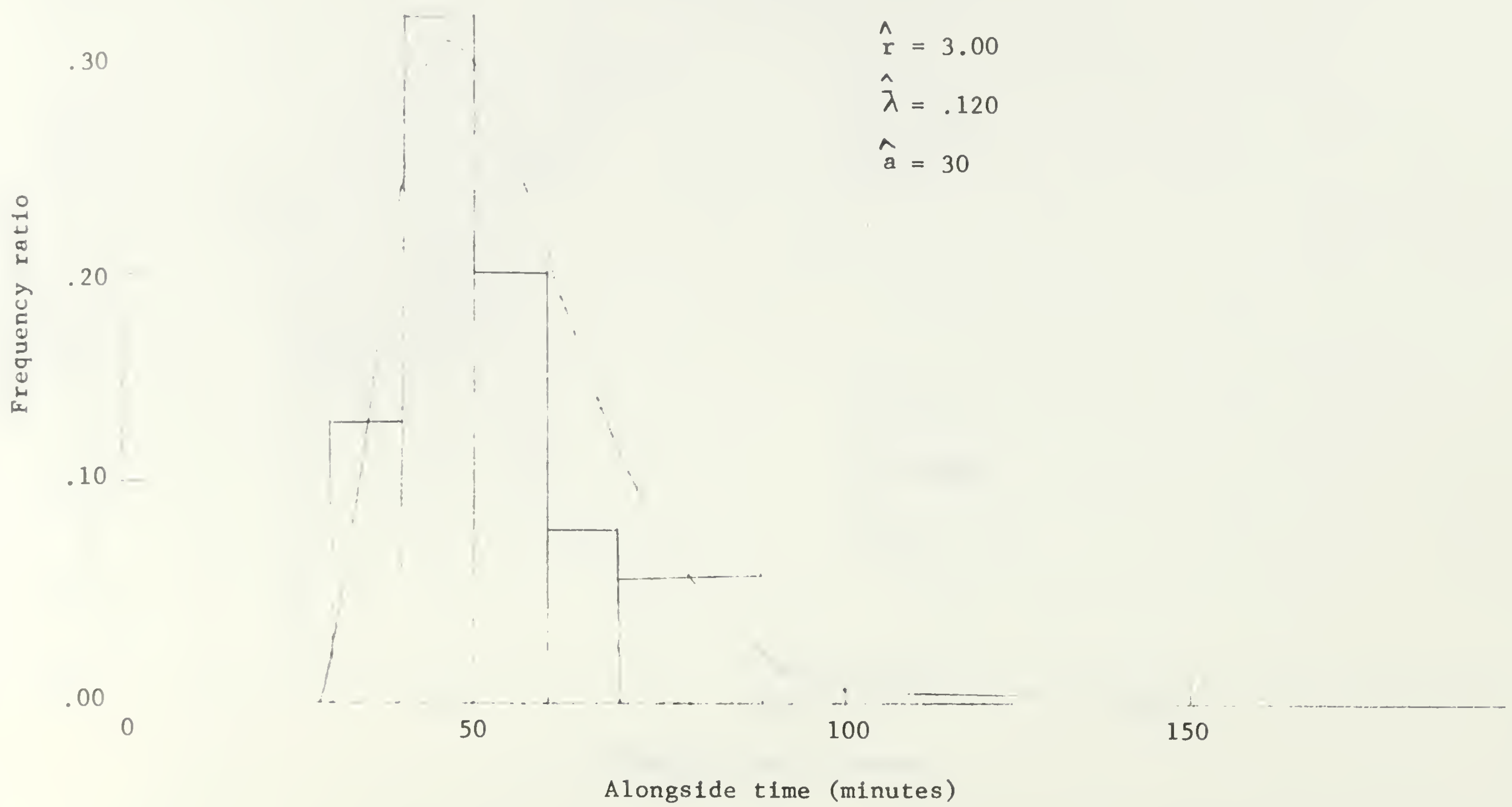


Figure 21. AO vs. DD - day. 1000-1099 barrels

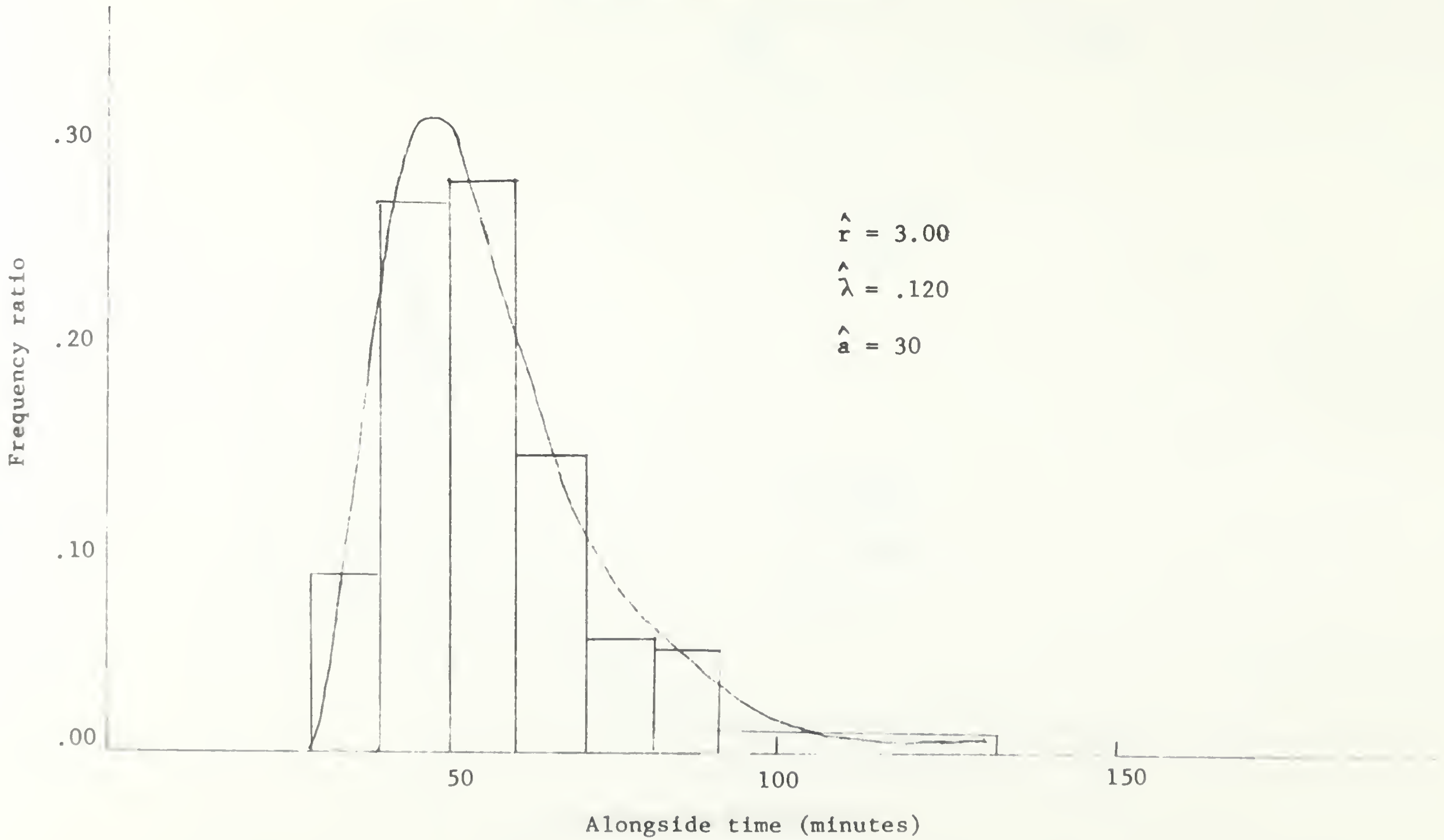


Figure 22. AO vs. DD - day. 1100-1199 barrels

Figure 23. OA vs. DD - day. 1200 - 1299 barrels

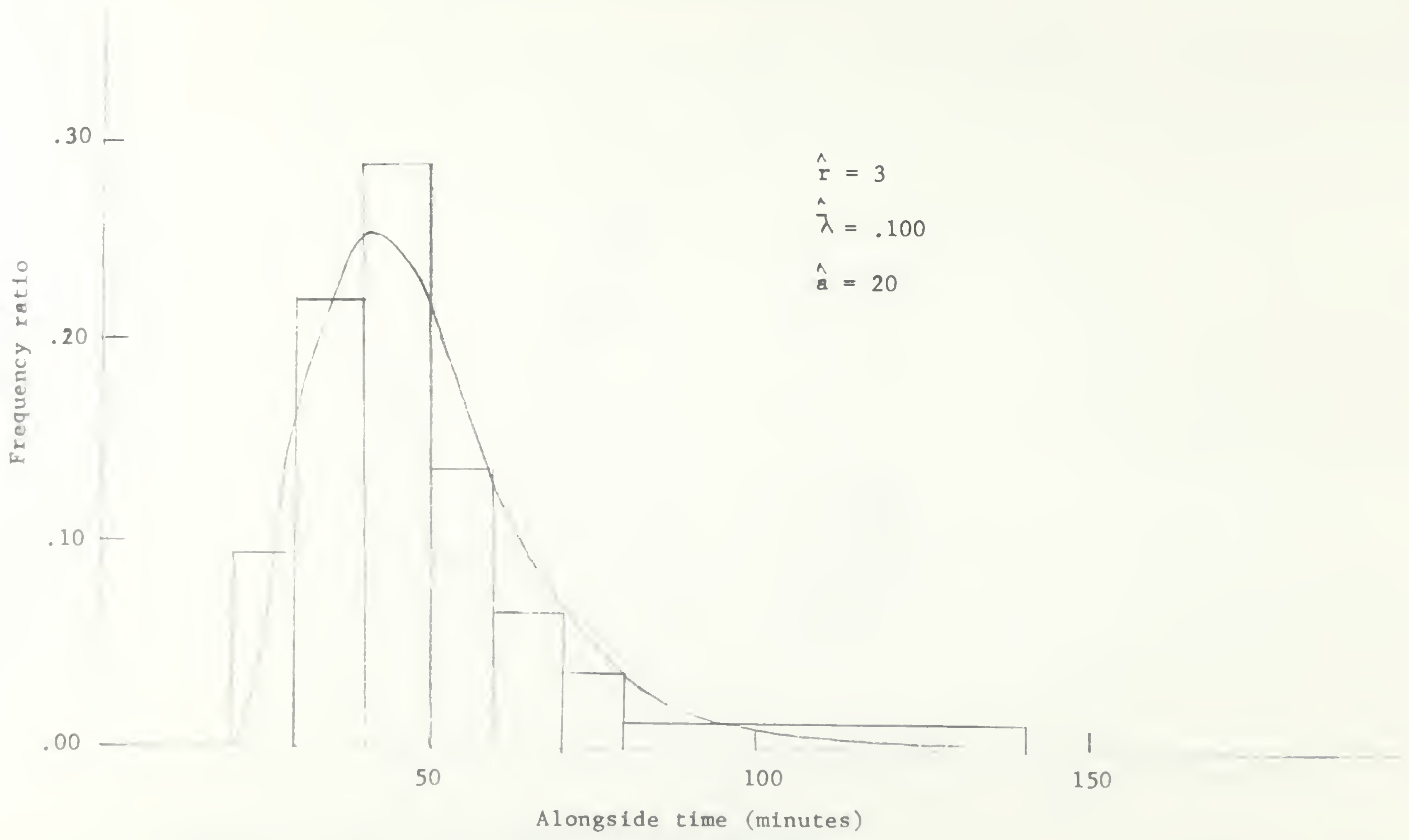


Figure 24. AO vs. DD - day. 1300-1399 barrels

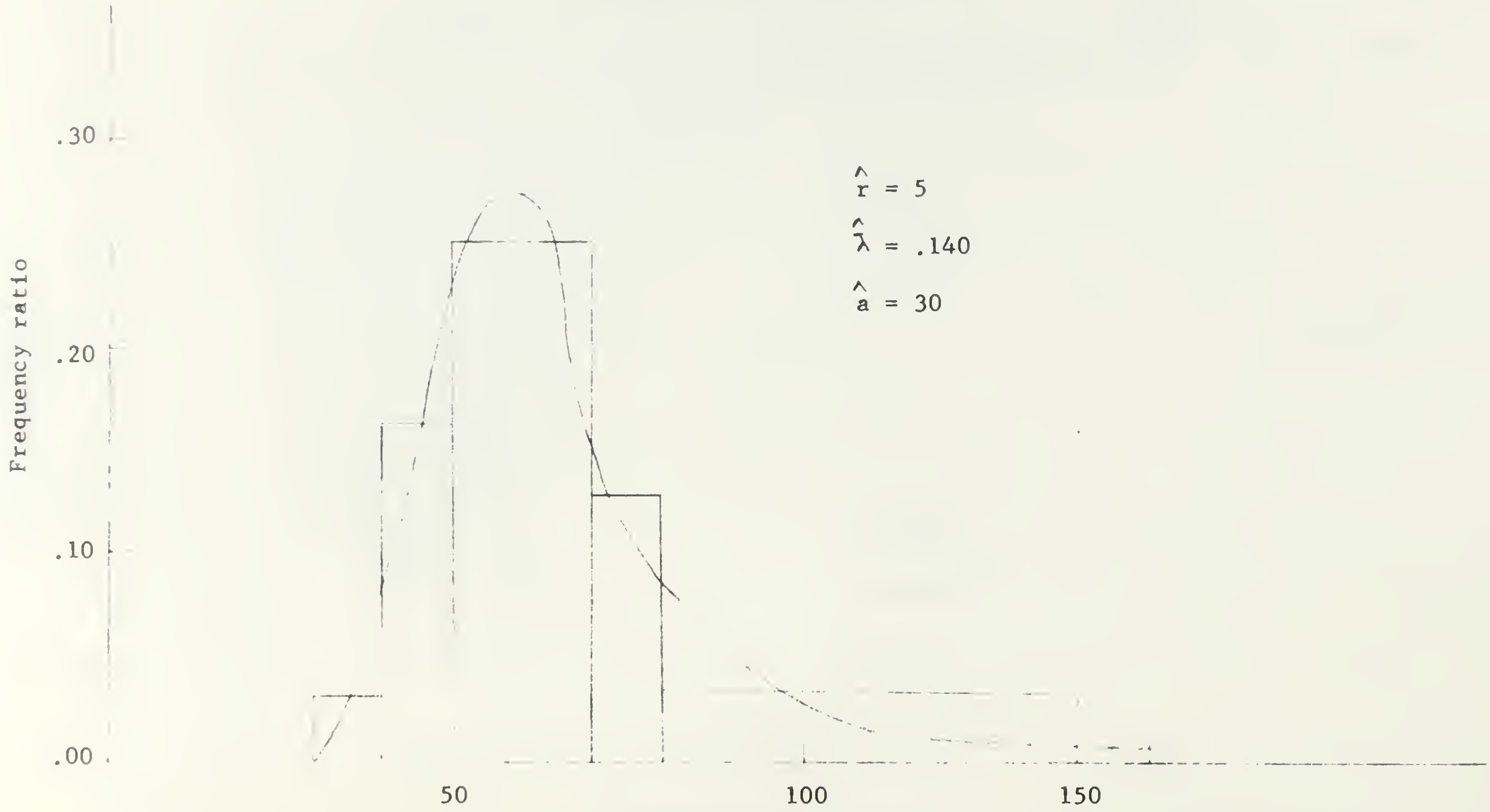


Figure 25. AO vs. DD - day. 1400 - 1499 barrels

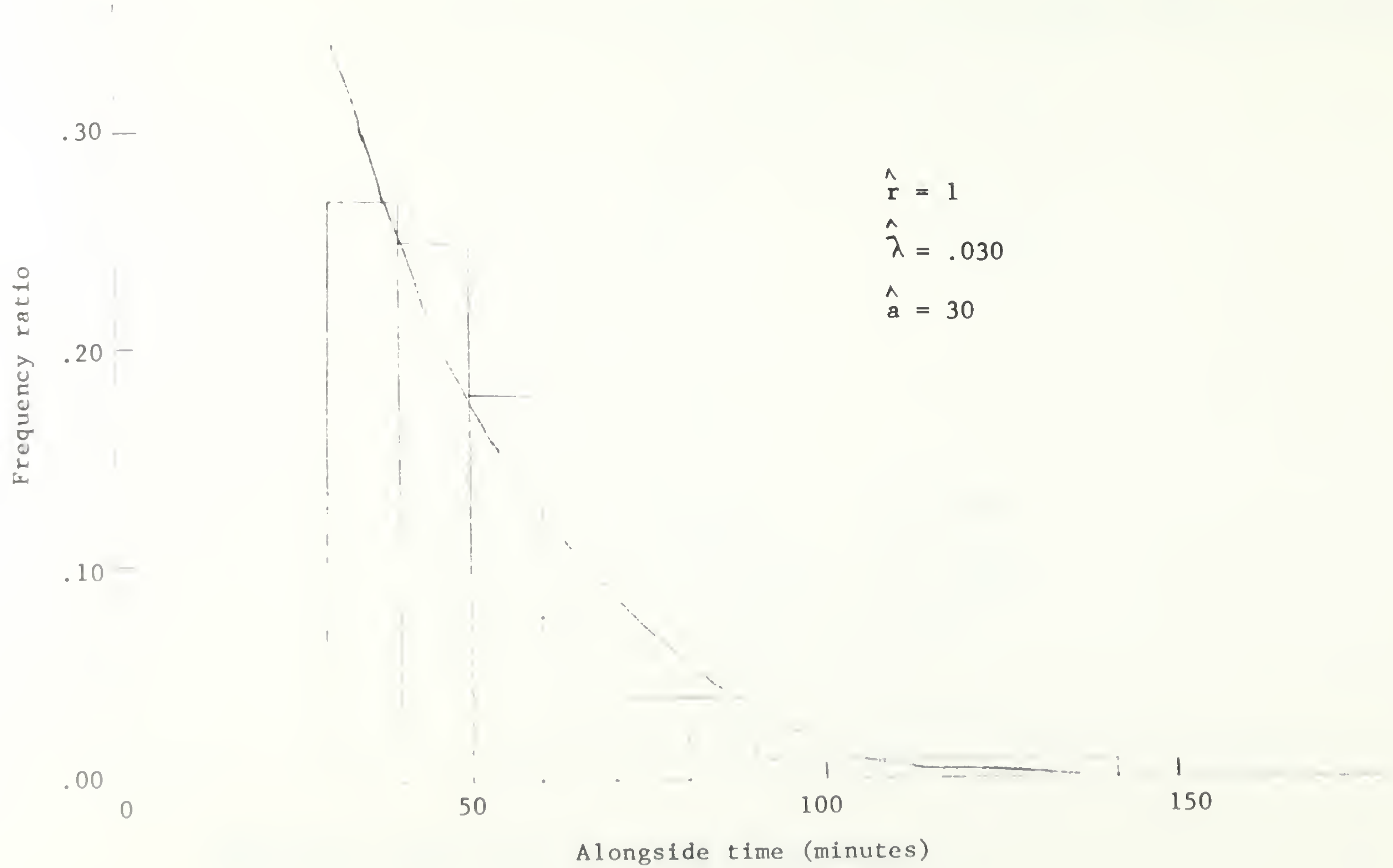


Figure 26. AO vs. DD - day. 1500 - 1599 barrels

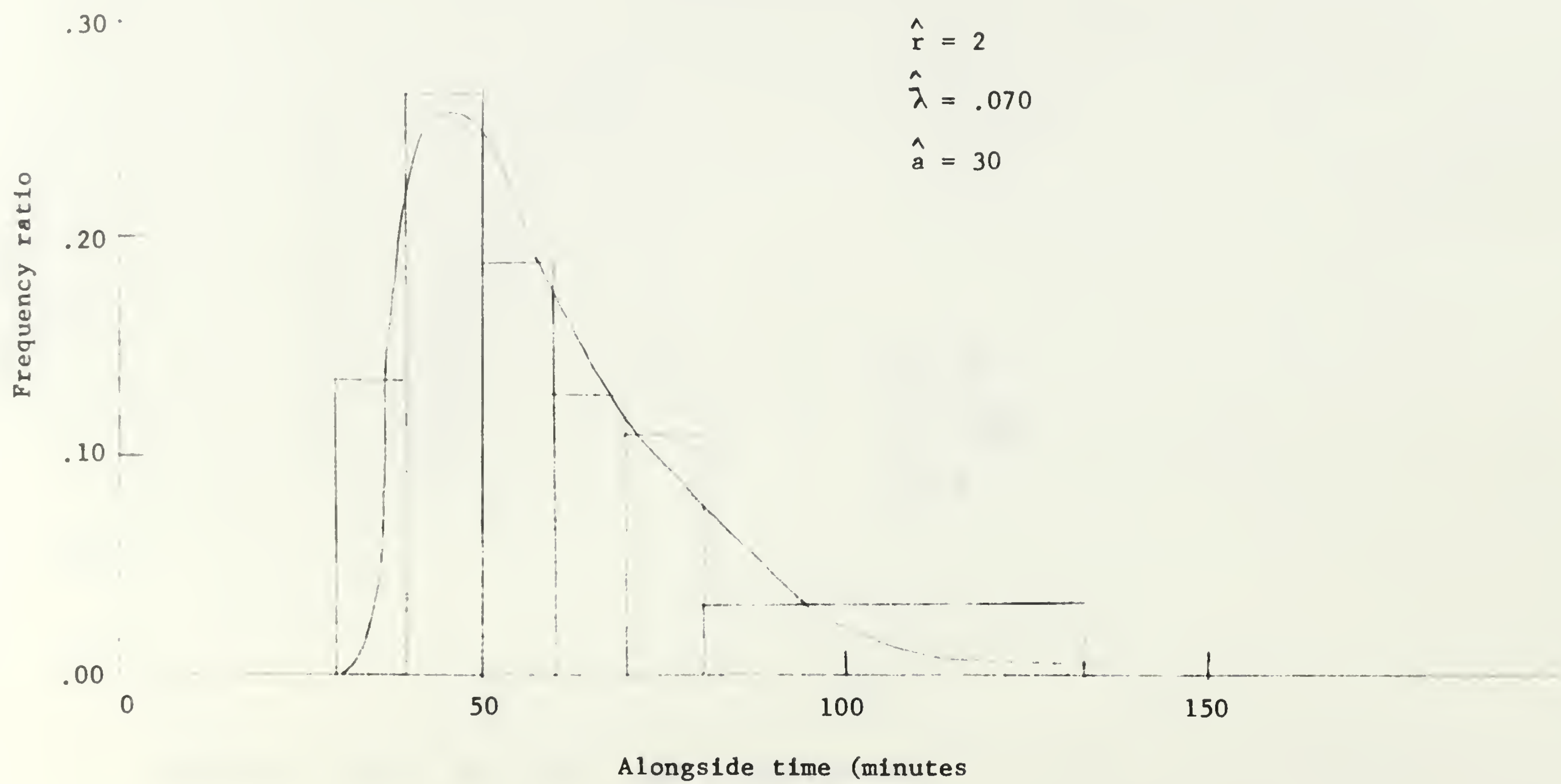


Figure 27. AO vs. DD - day. 1600 - 1699 barrels

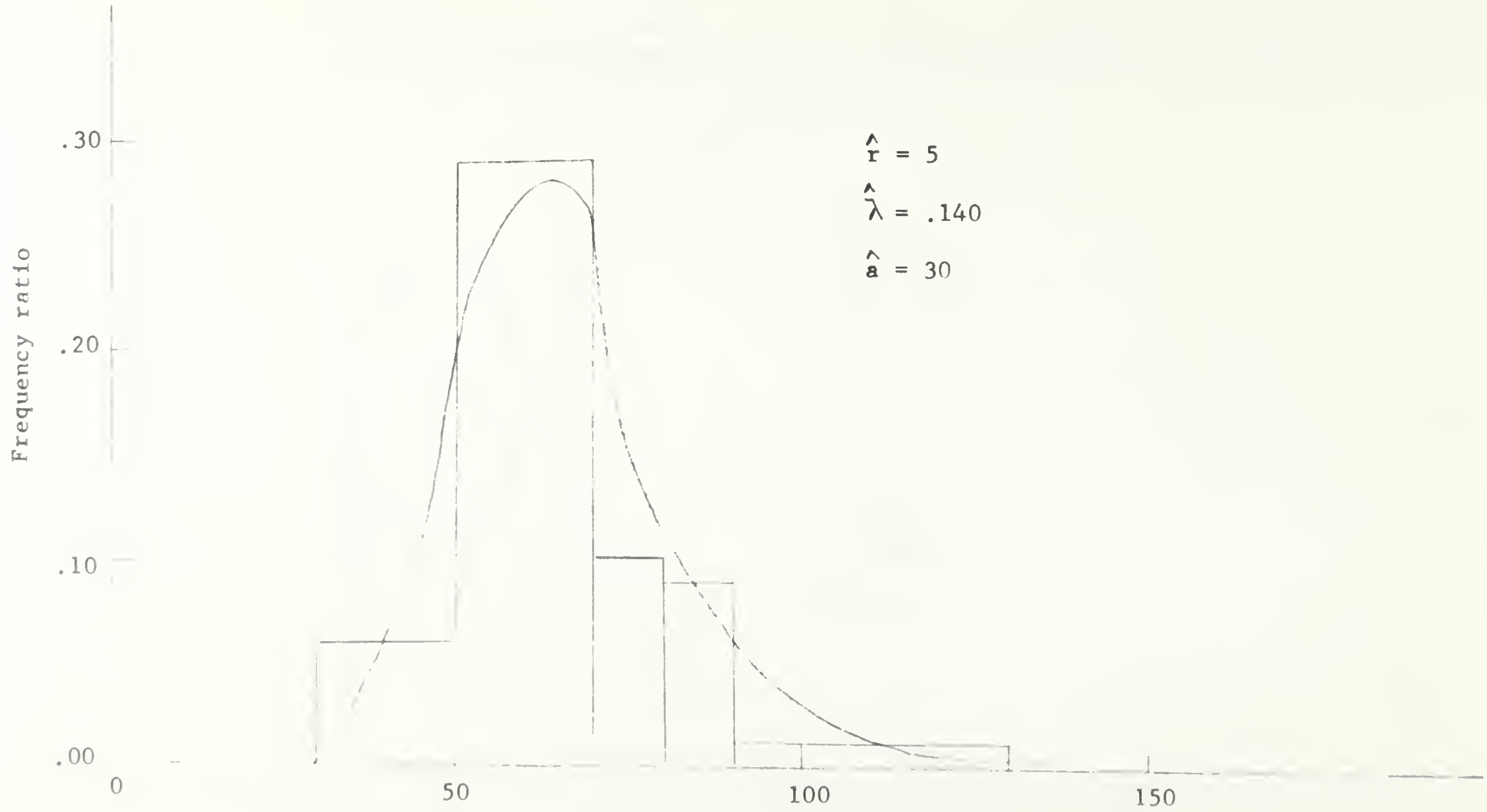


Figure 28. AO vs. DD - day. 1700 - 1799 barrels

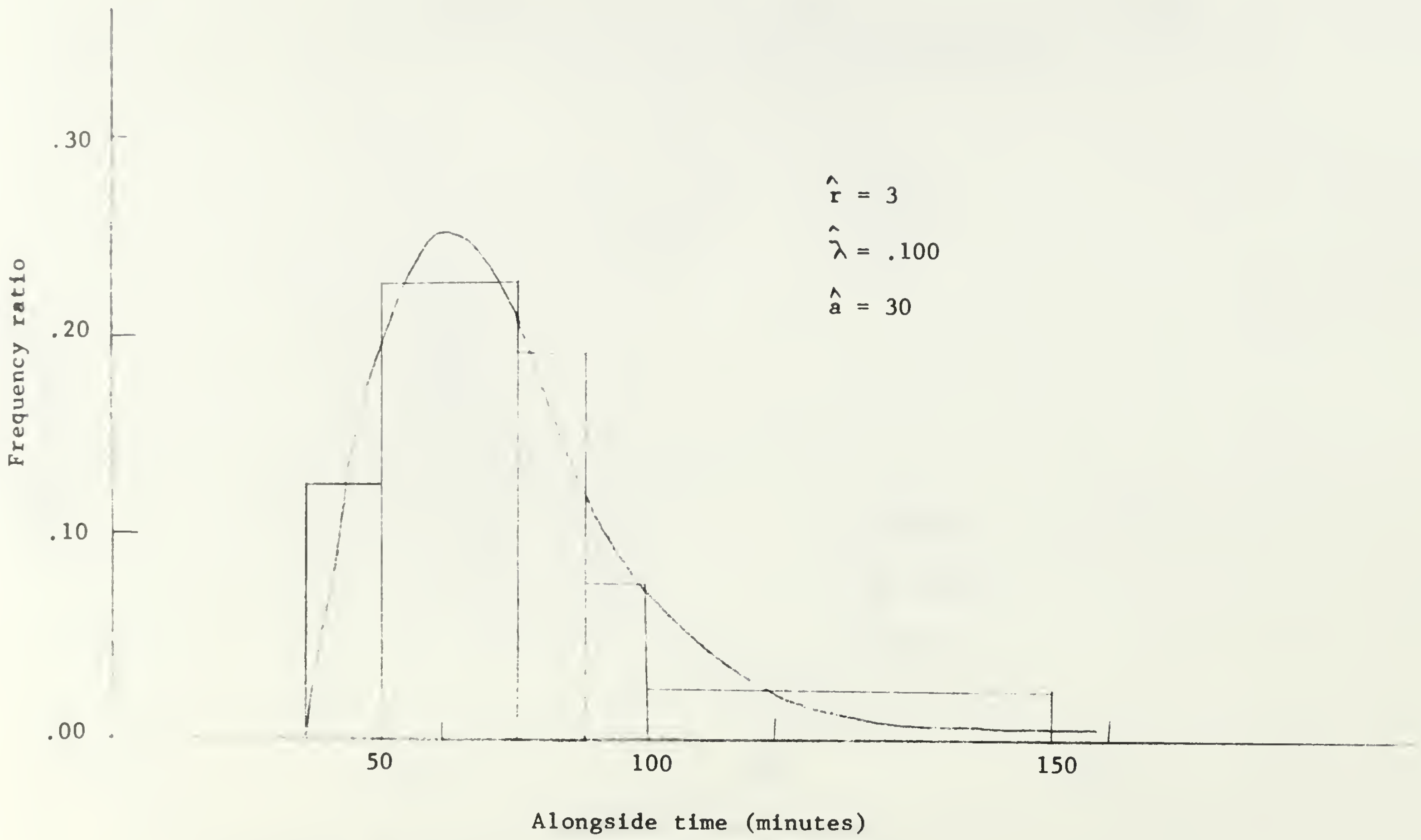


Figure 29. AO vs. DD - day. 1800 - 1899 barrels

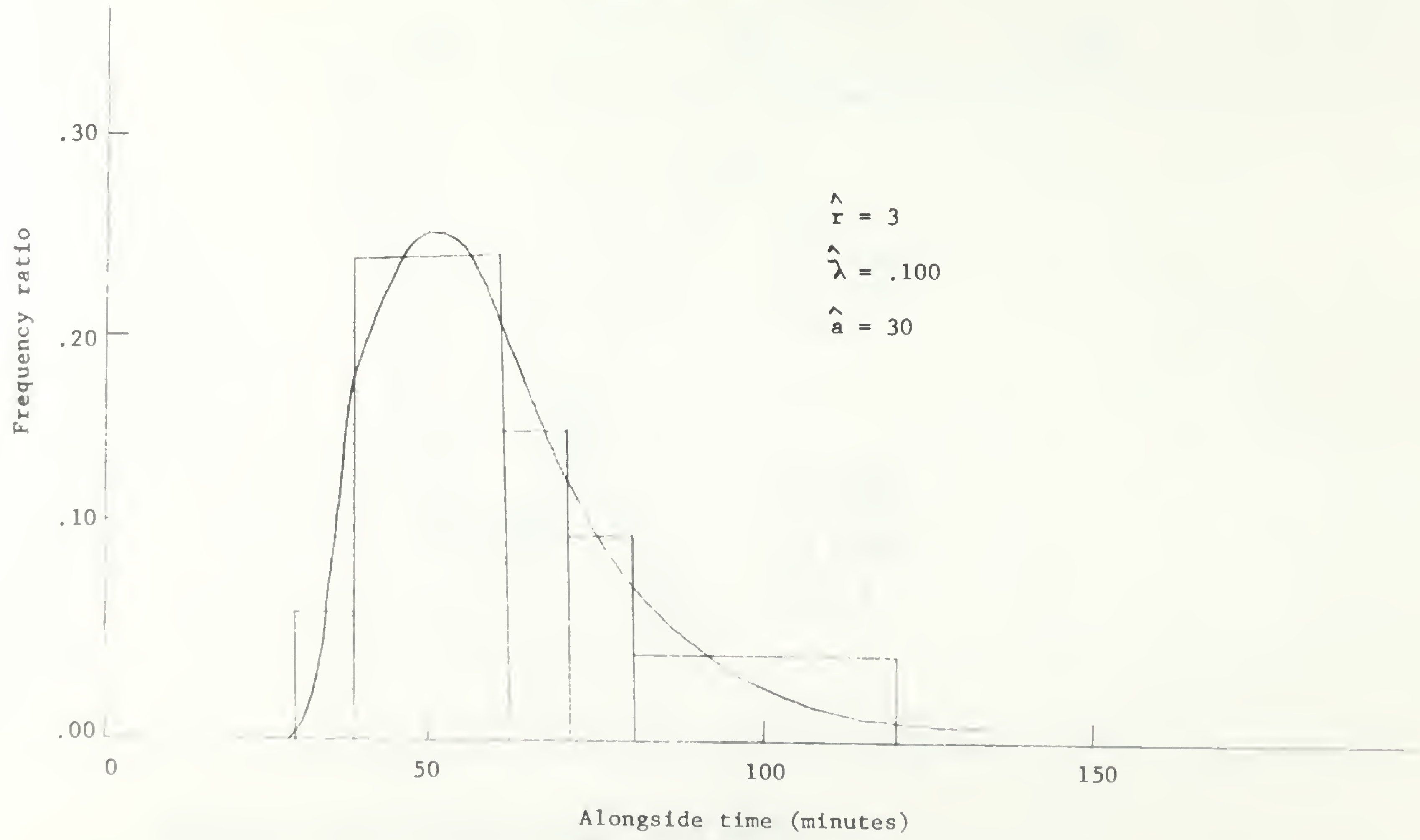


Figure 30. AO vs. DD - day. 1900 - 1999 barrels

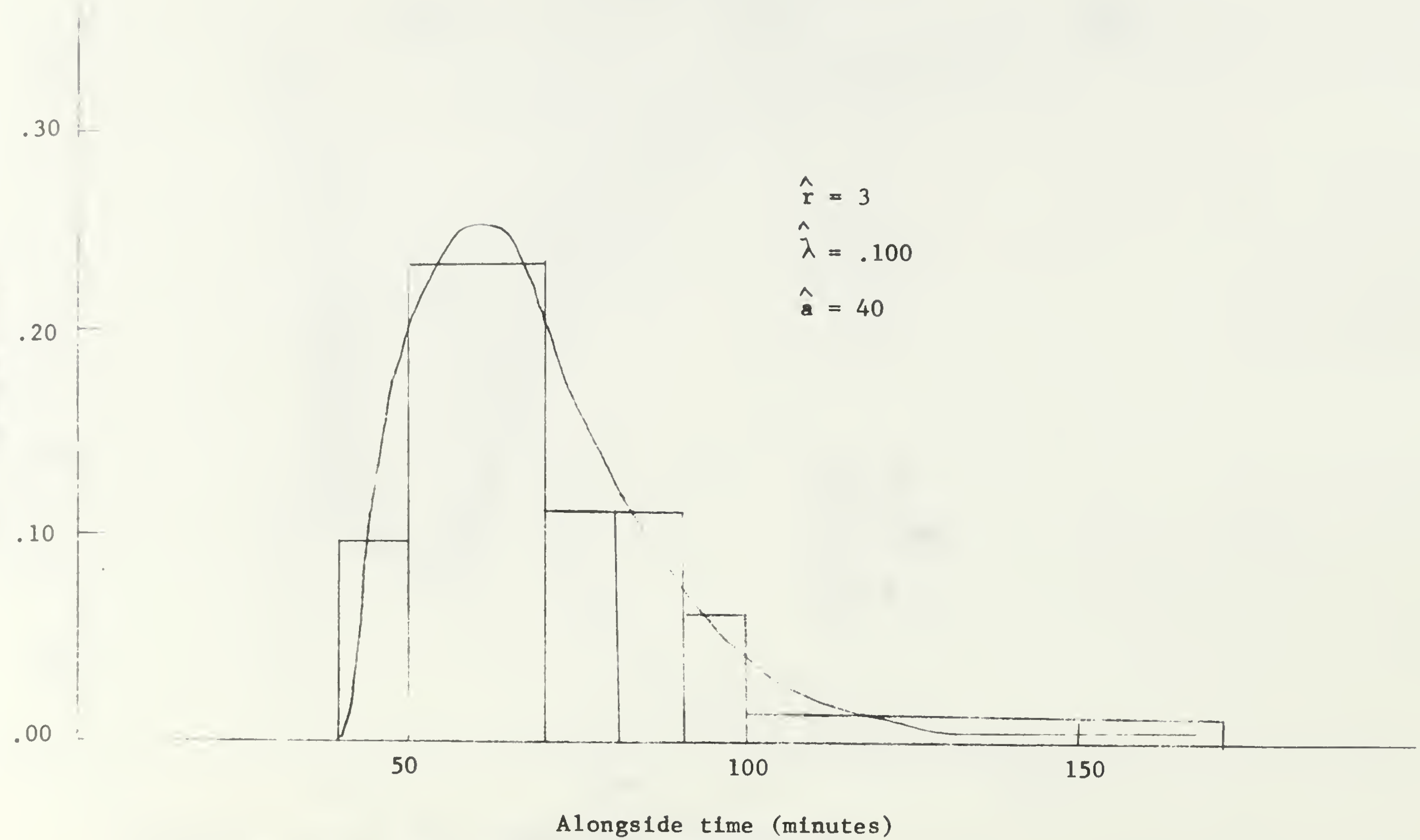


Figure 31. AO vs. DD - day. 2000 - 2099 barrels

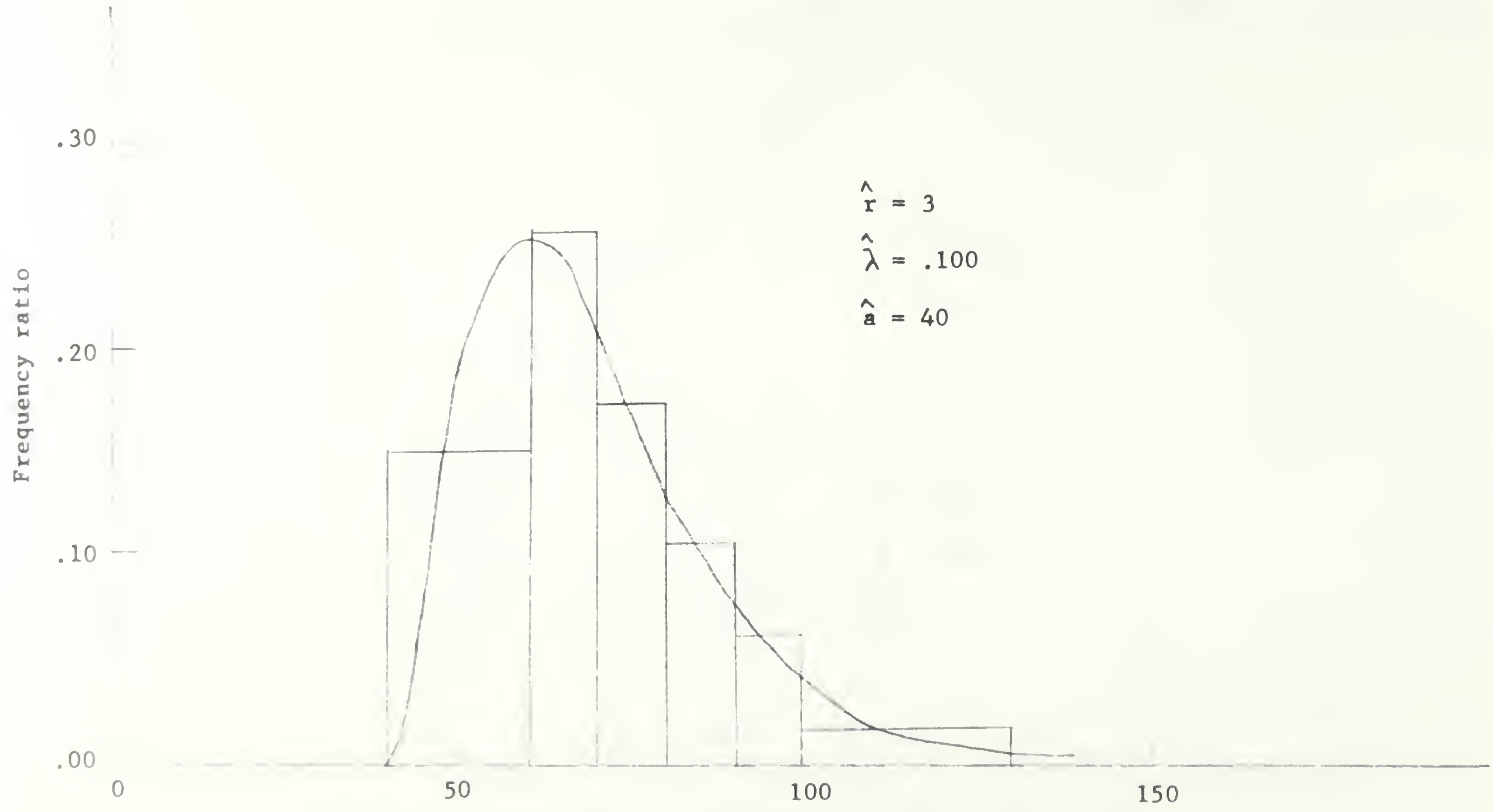


Figure 32. AO vs. DD - day. 2100 - 2199 barrels

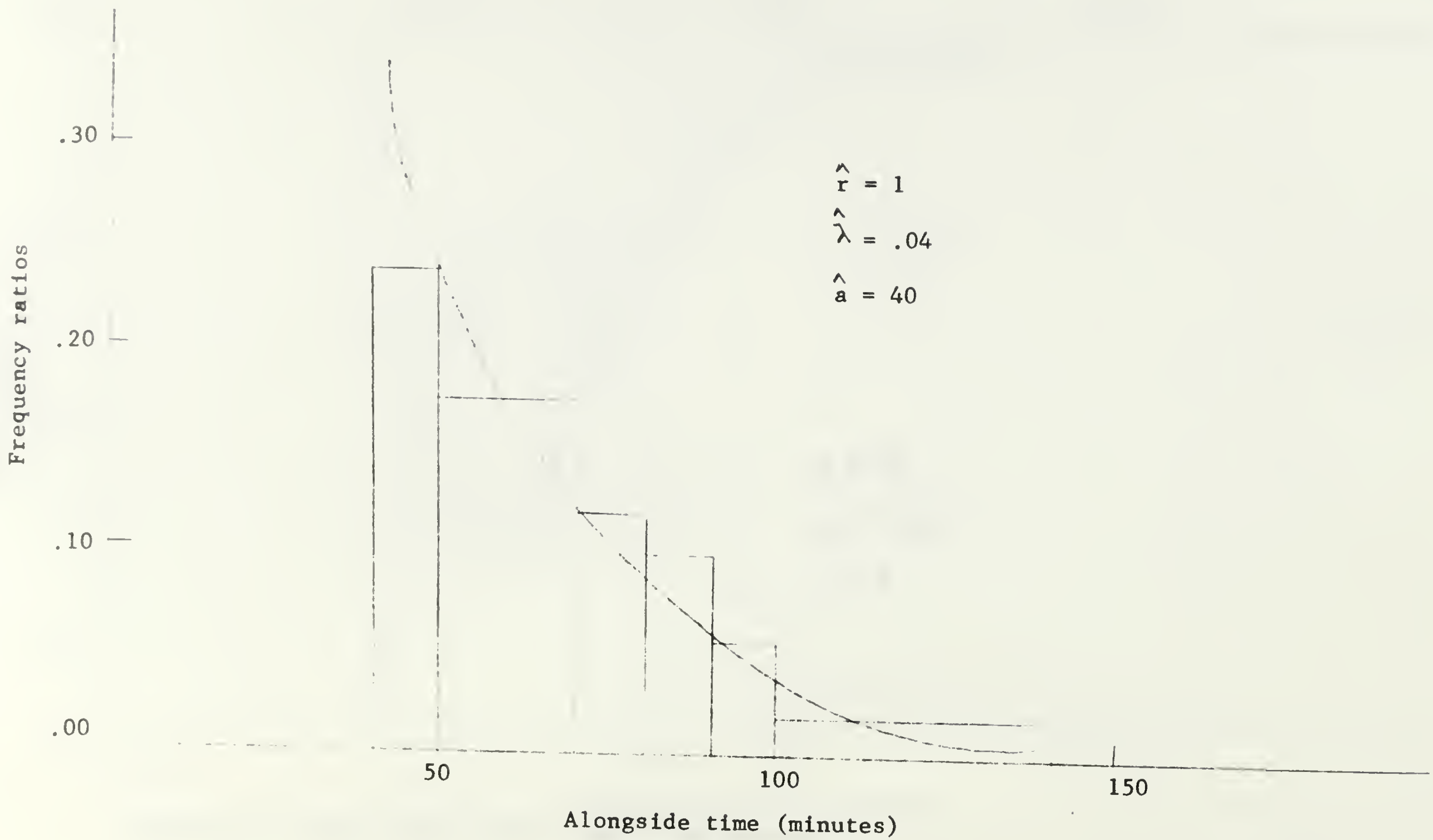


Figure 33. AO vs. DD - day. 2200 - barrels

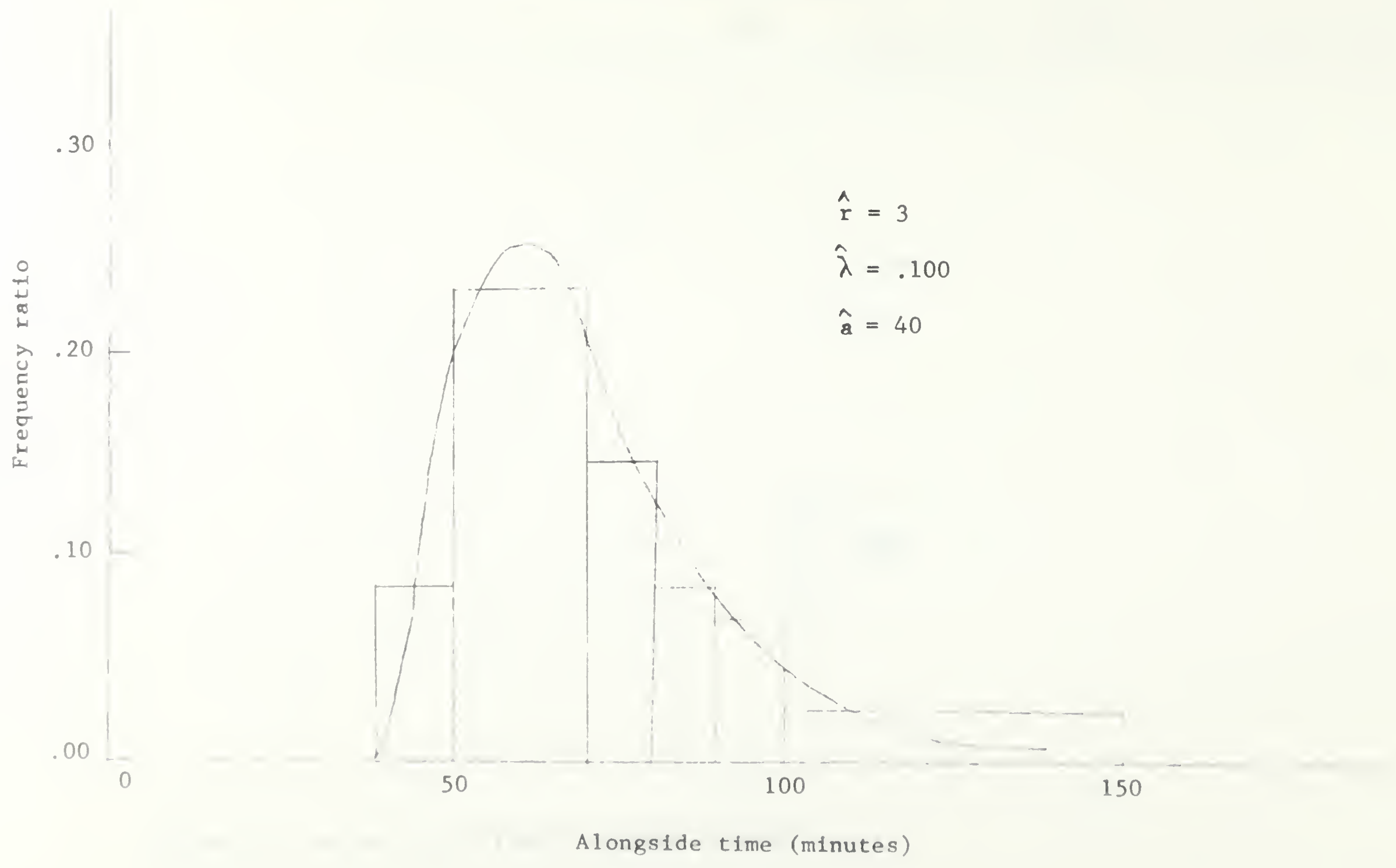


Figure 34. AO vs. DD - night. 0 - 600 barrels

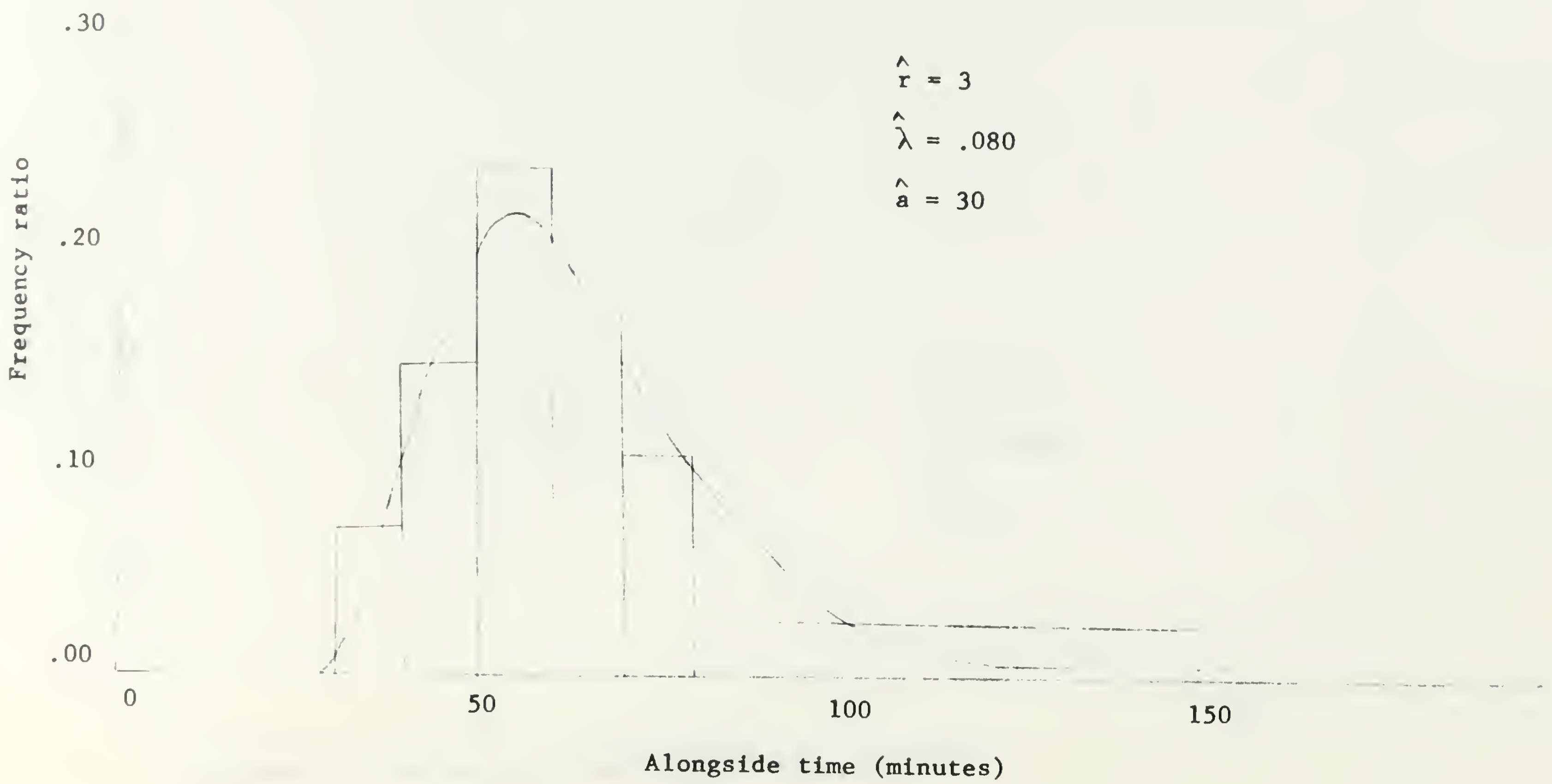


Figure 35. AO vs. DD - night. 600 - 699 barrels

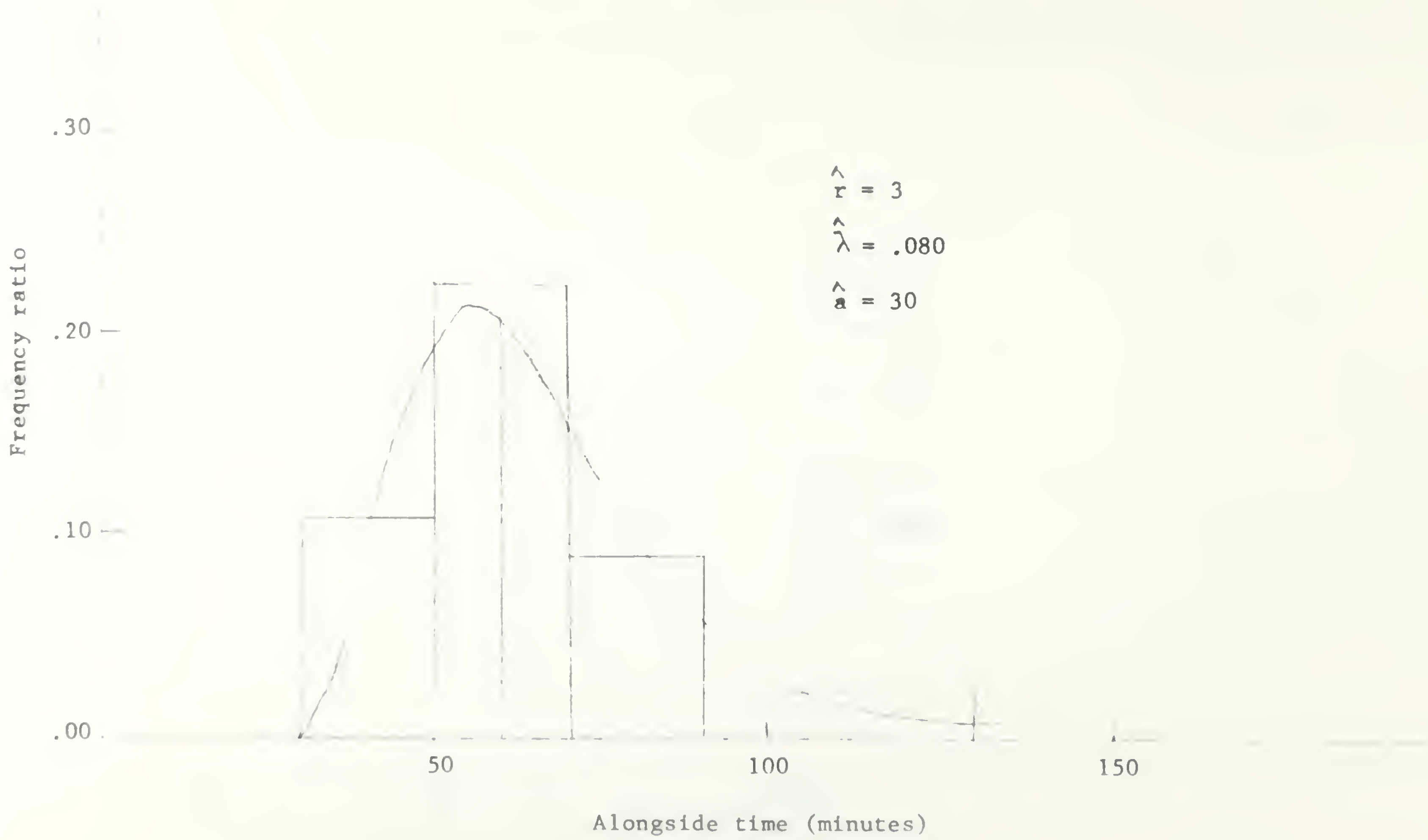


Figure 36. AO vs. DD - night. 700 - 799 barrels

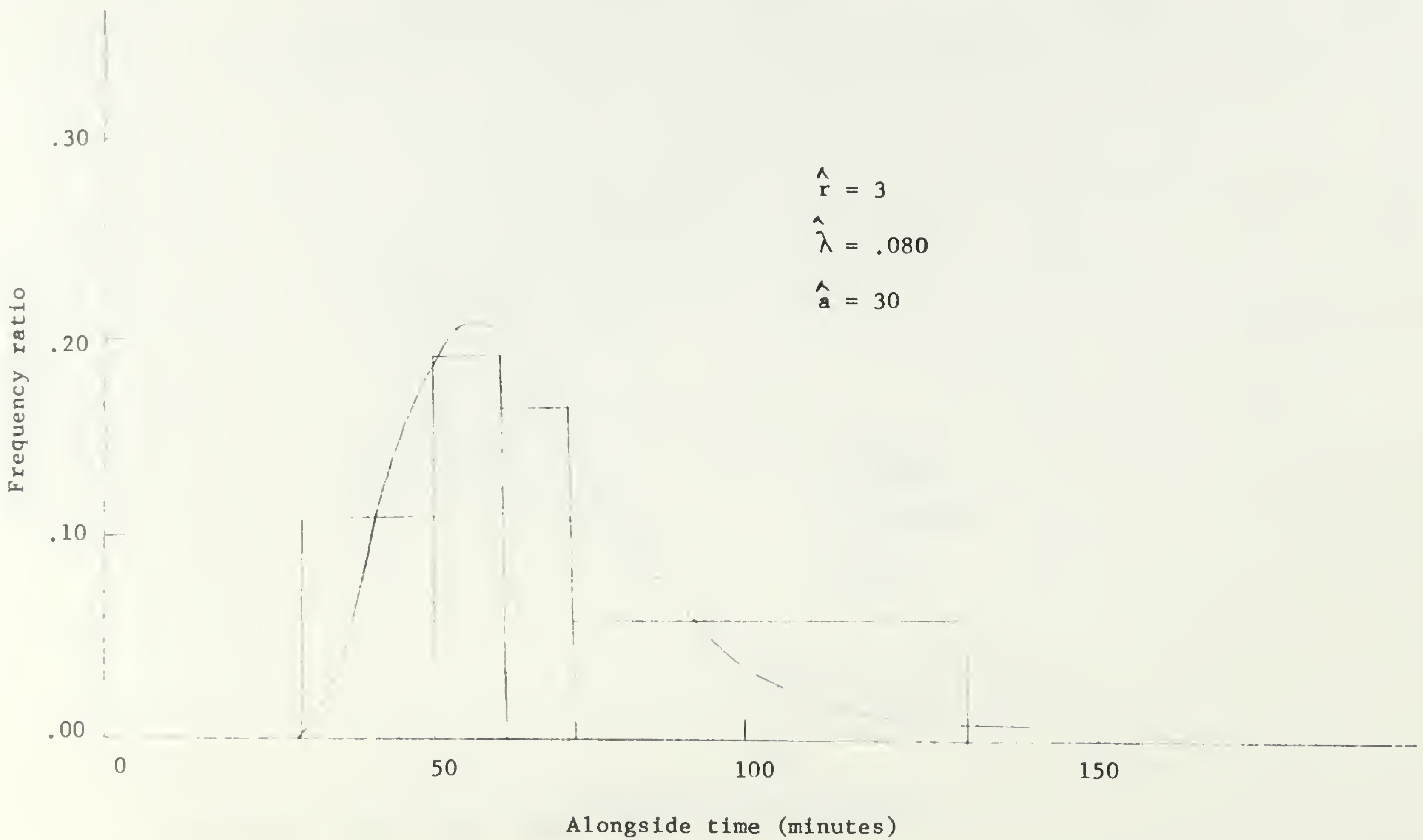


Figure 37. AO vs. DD - night. 800 - 899 barrels

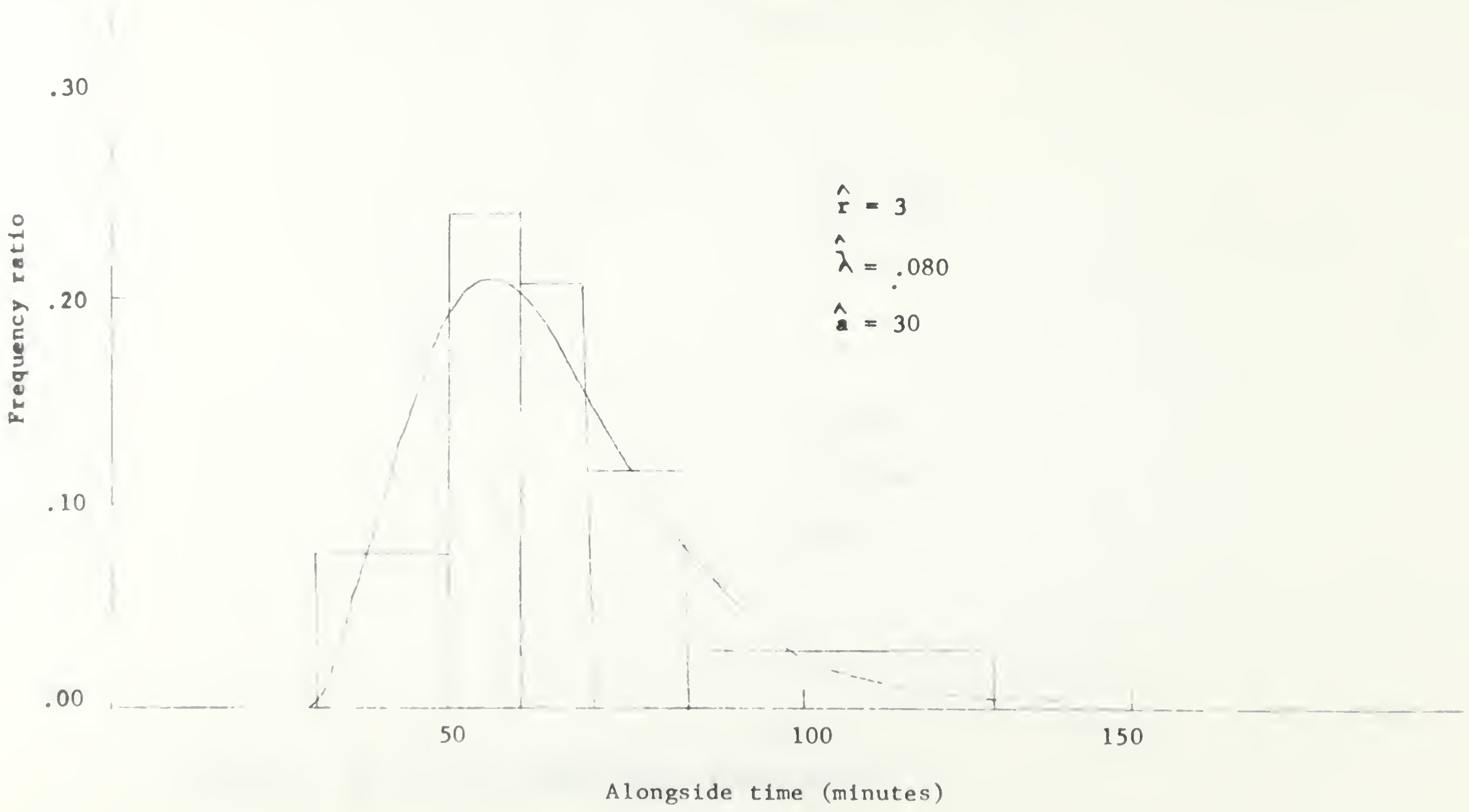


Figure 38. AO vs. DD - night. 900 - 999 barrels

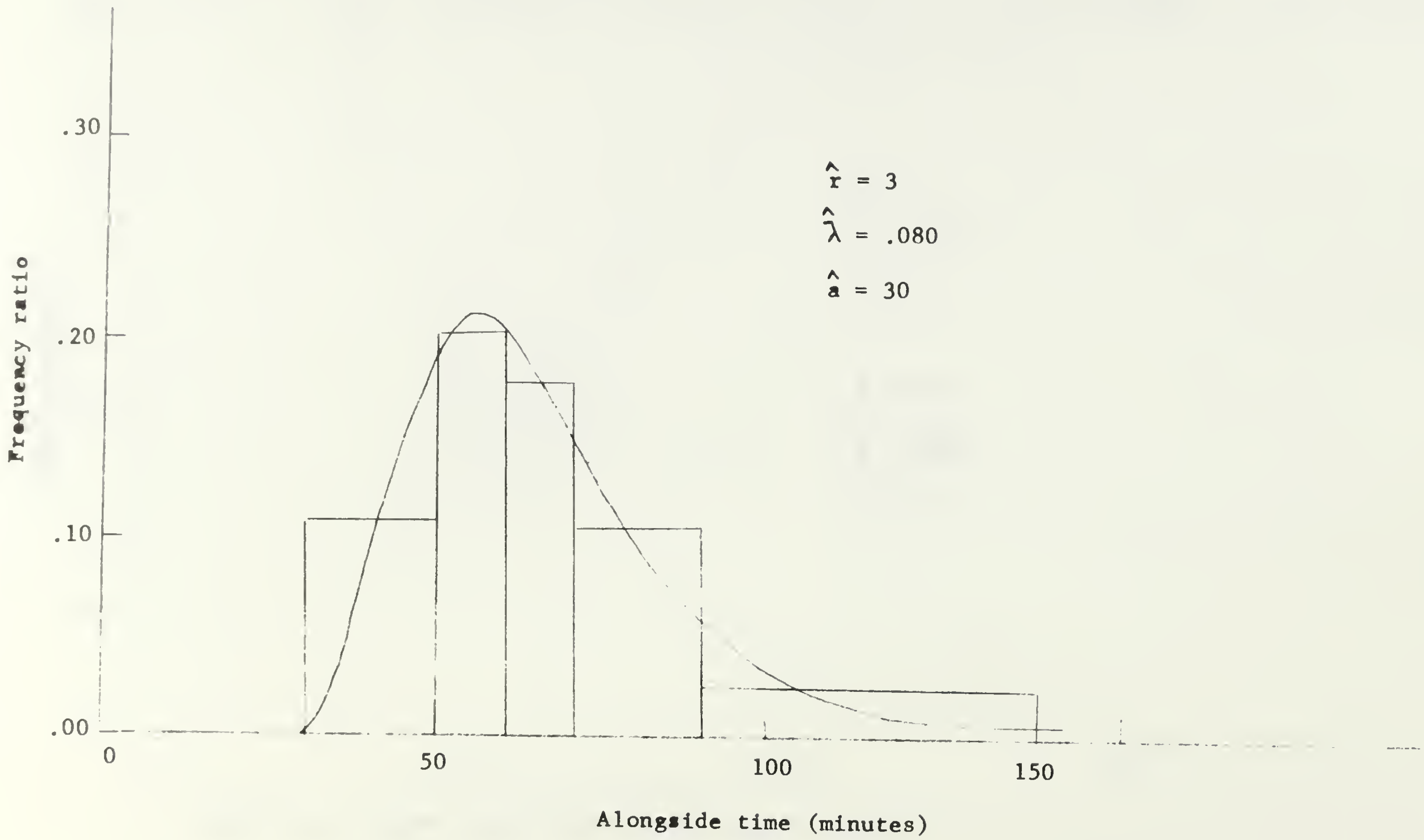


Figure 39. AO vs. DD - night. 1000 - 1099 barrels

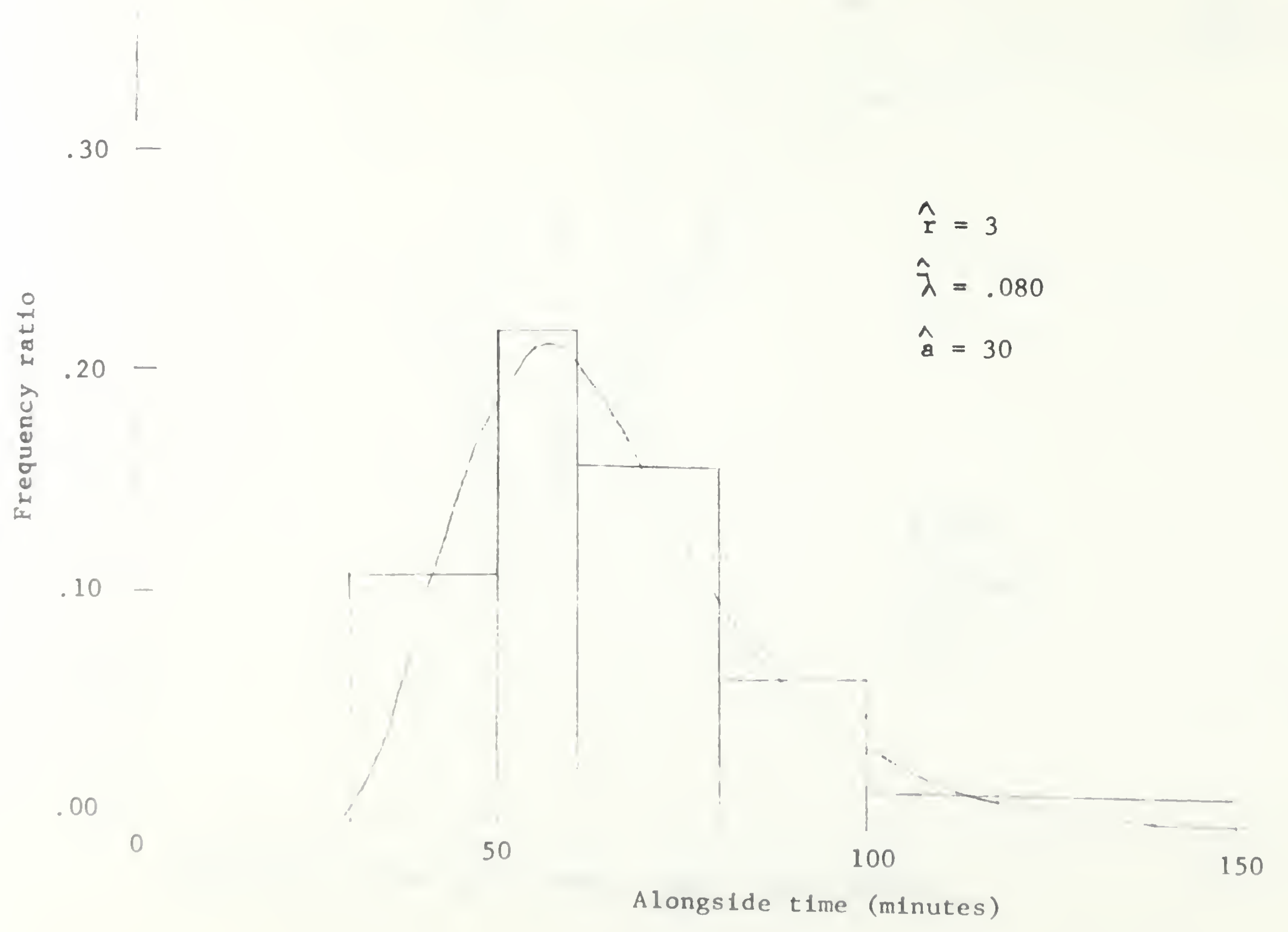


Figure 40. AO vs. DD - night. 1100 - 1199 barrels

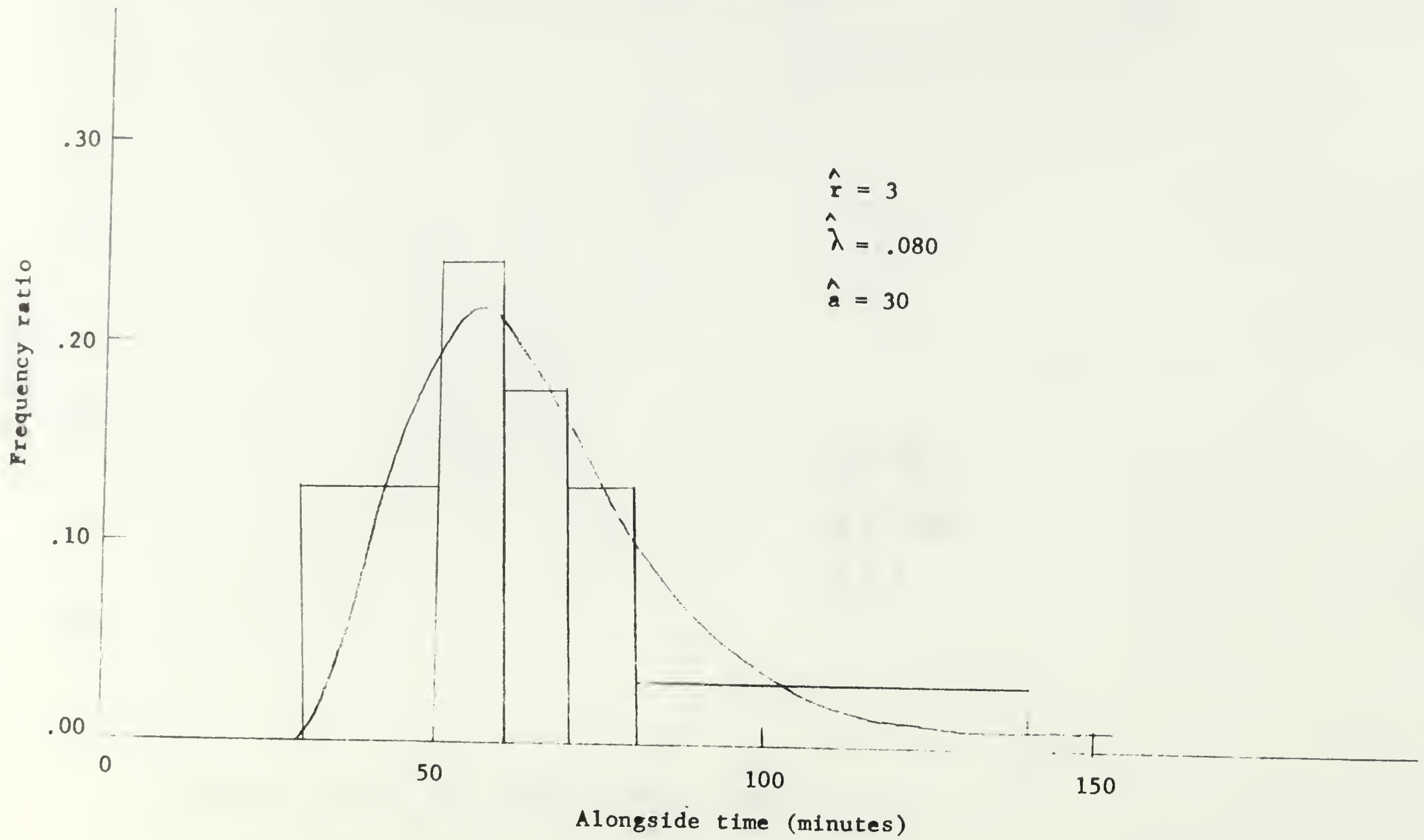


Figure 41. AO vs. DD - night. 1200 - 1299 barrels

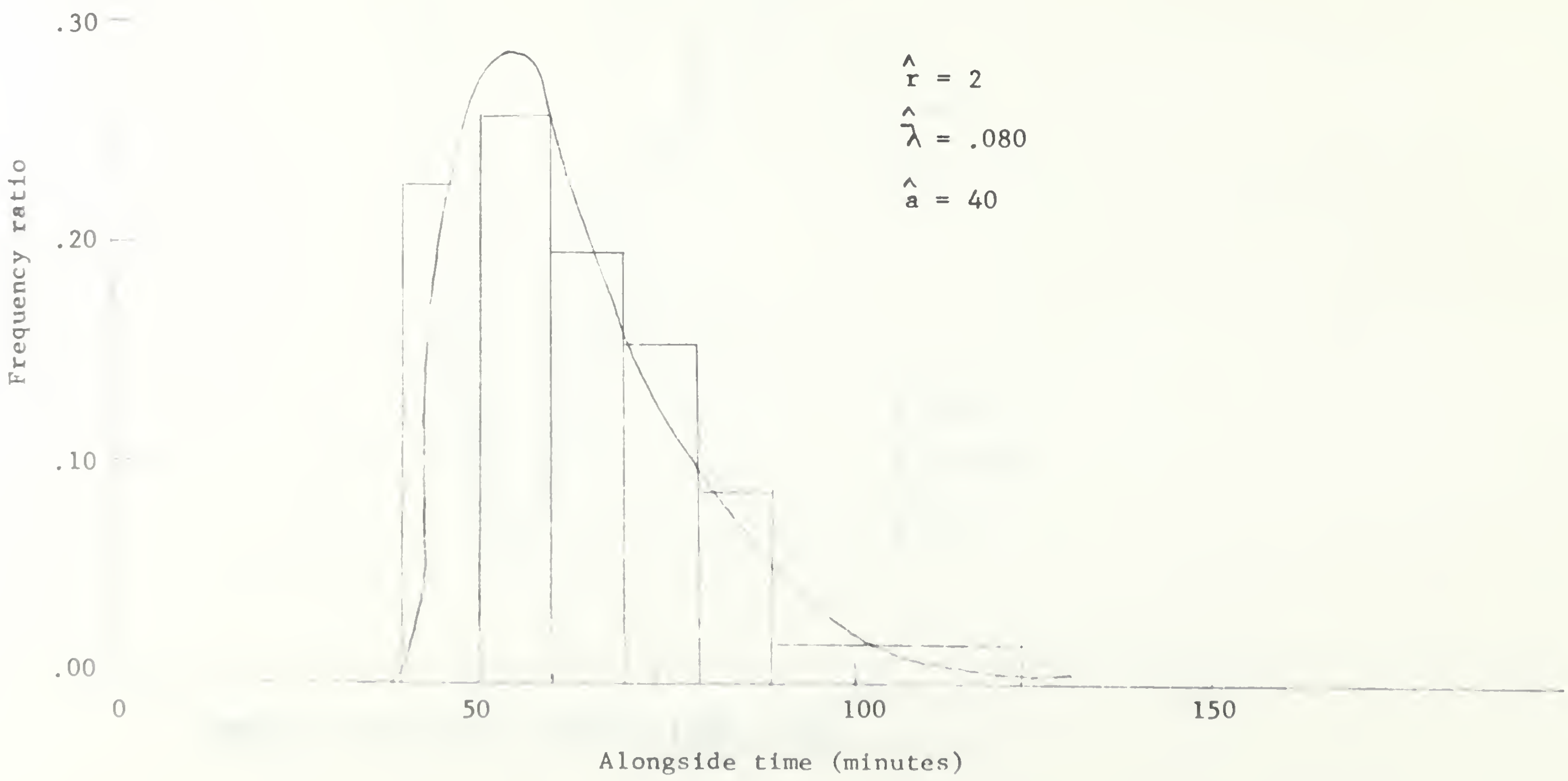


Figure 42. AO vs. DD - night. 1300 - 1399

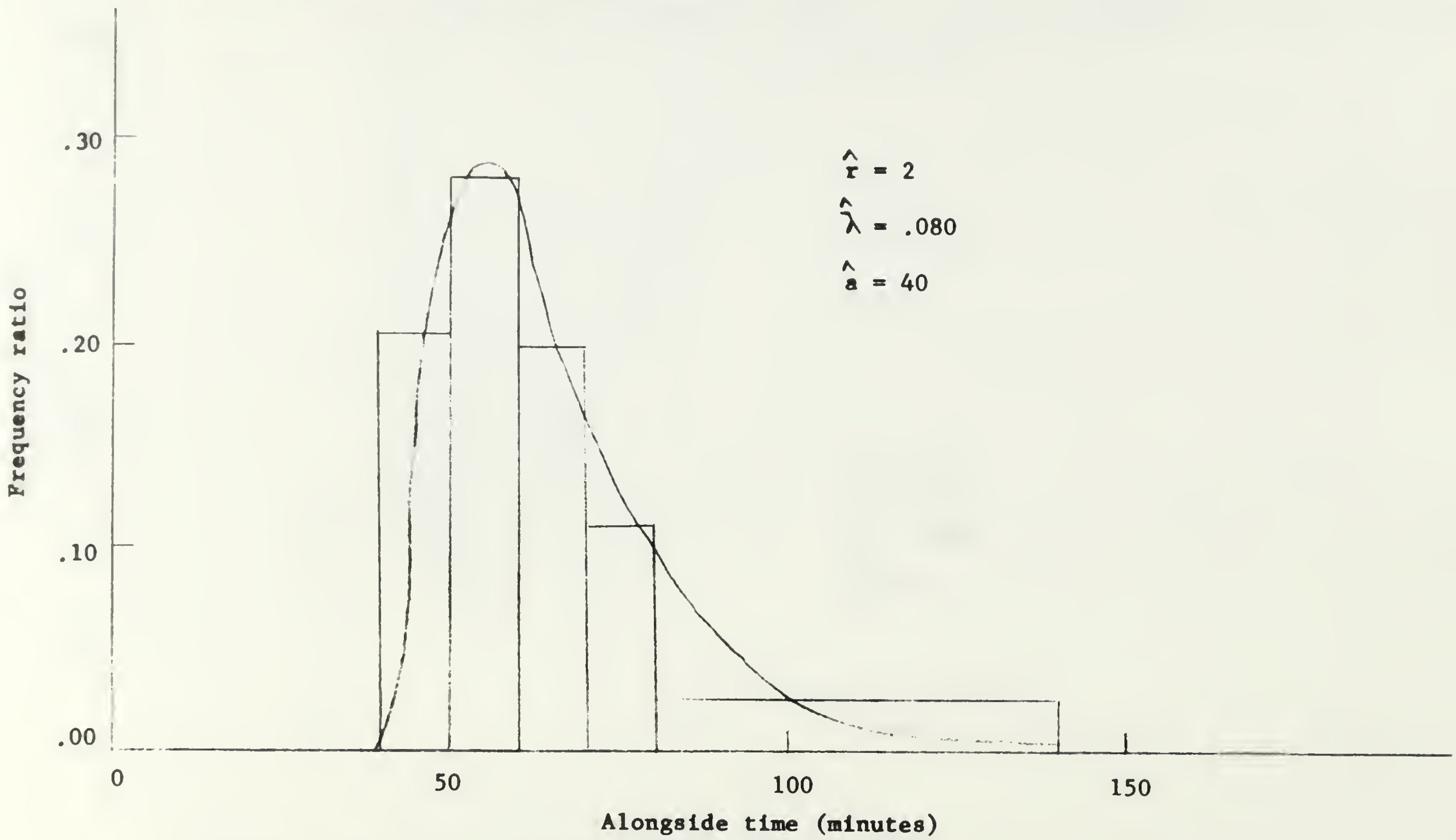


Figure 43. AO vs. DD - night. 1400 - 1499 barrels

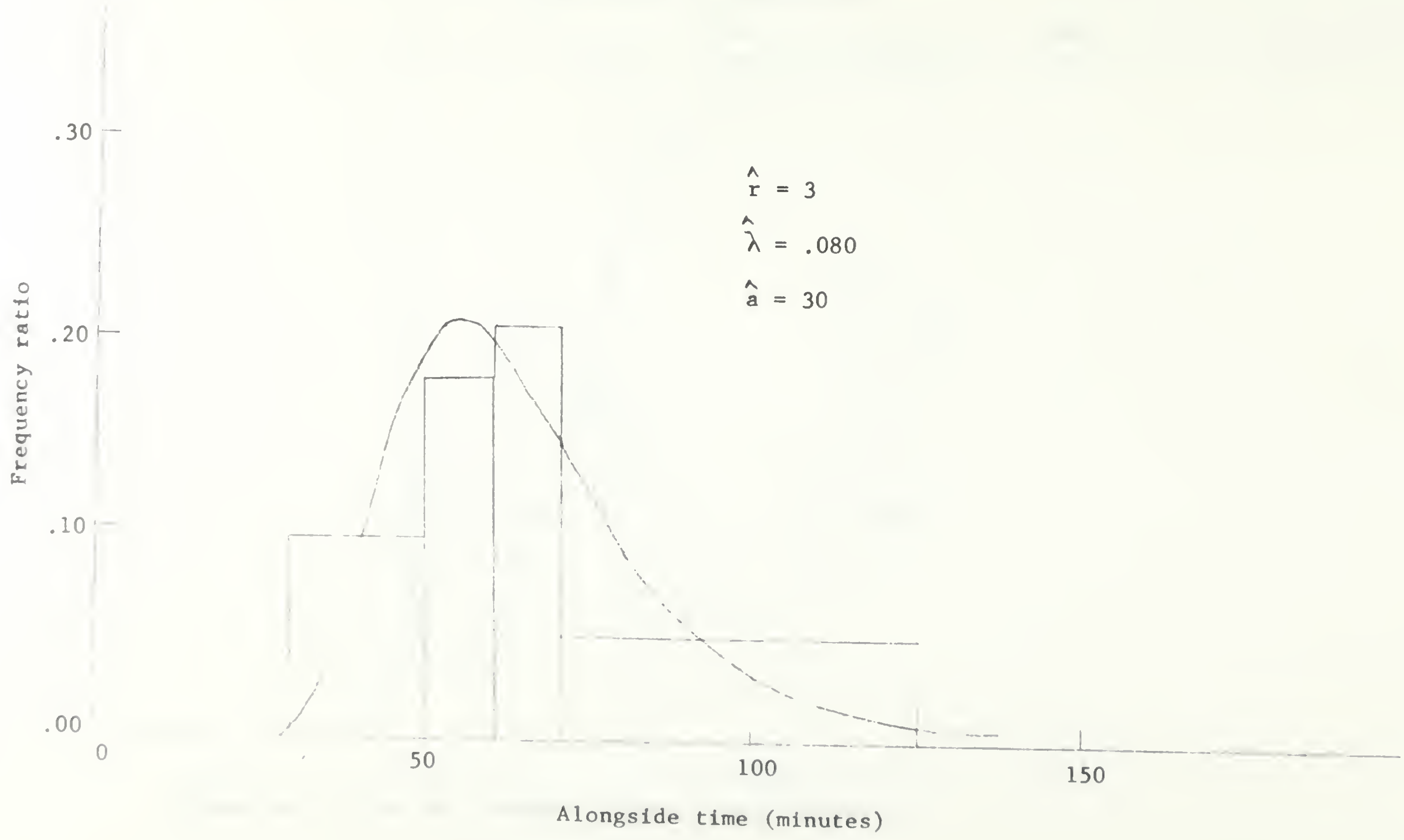


Figure 44. AO vs. DD - night. 1500 - 1599 barrels

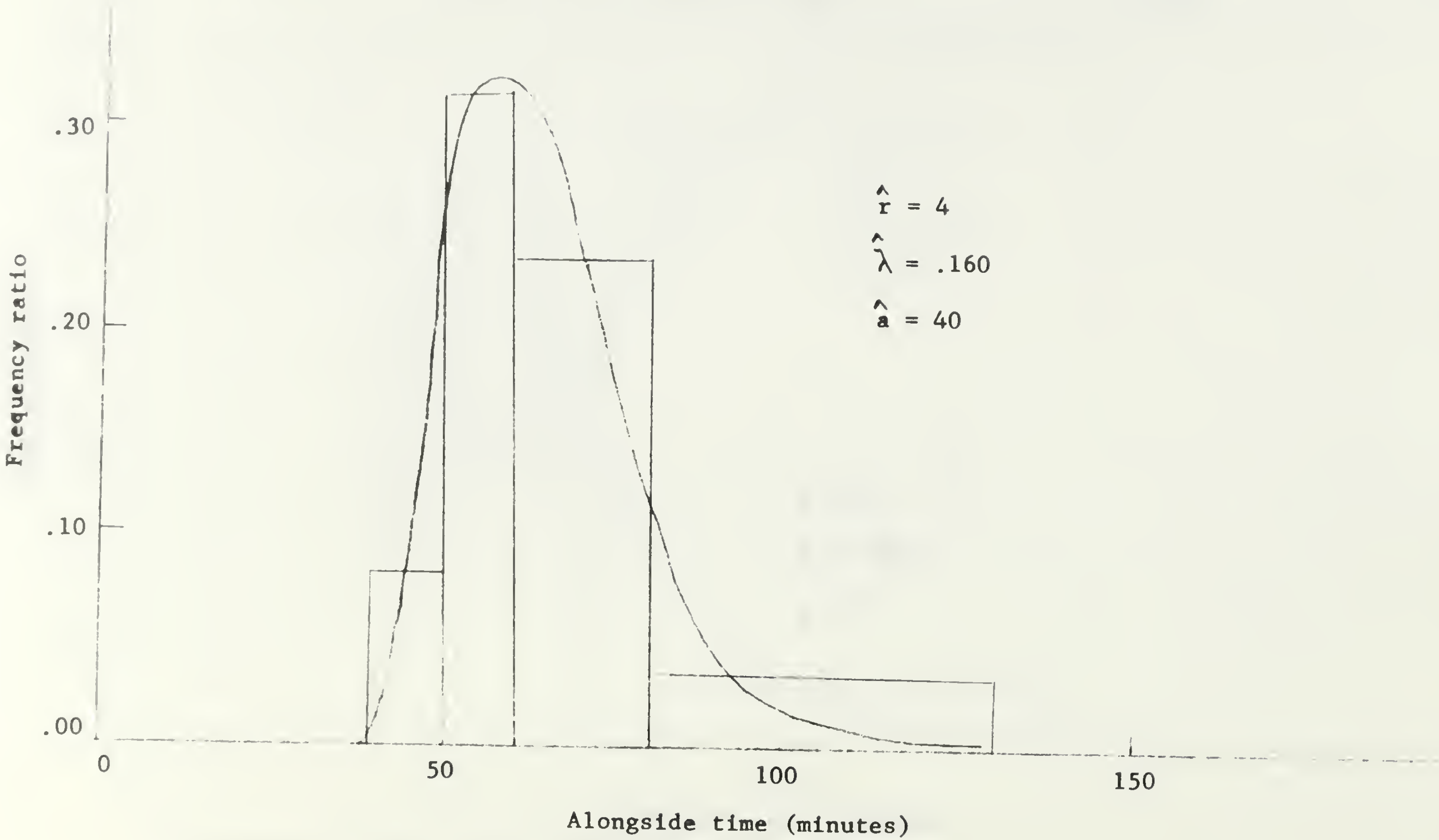


Figure 45. AO vs. DD - night. 1600 - 1699 barrels

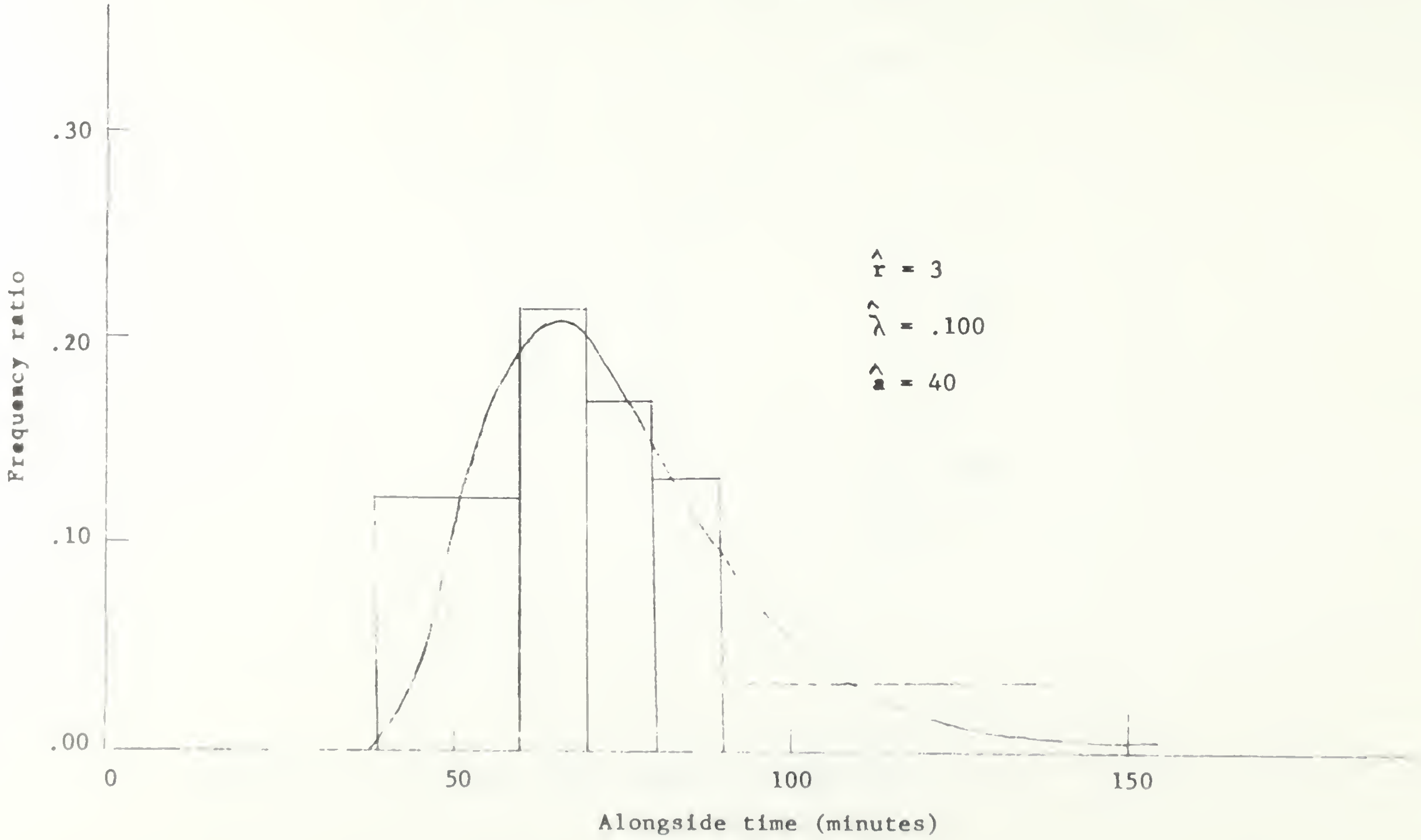


Figure 46. AO vs. DD - night. 1700 - 1799 barrels

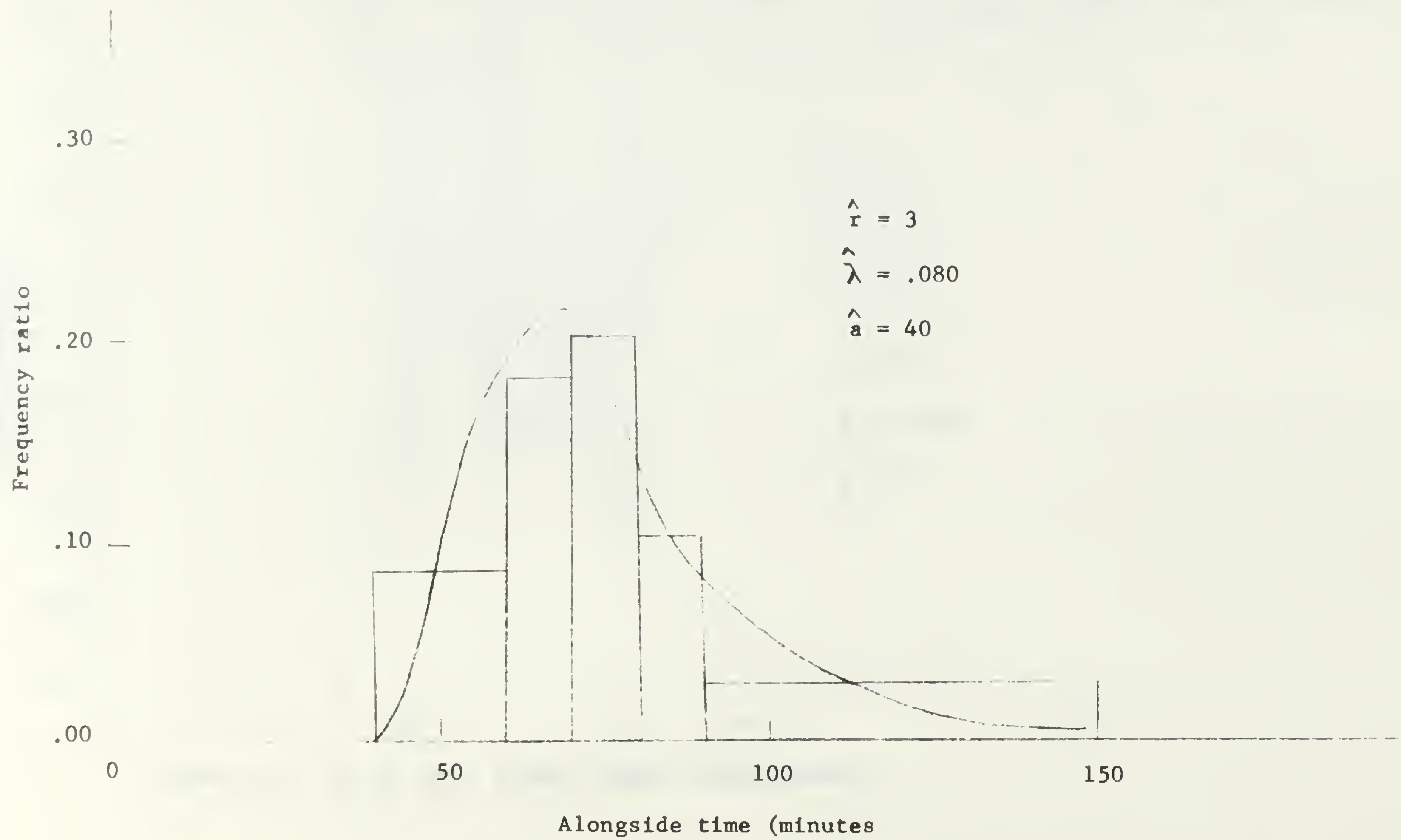


Figure 47. AO vs. DD - night. 1800 - barrels

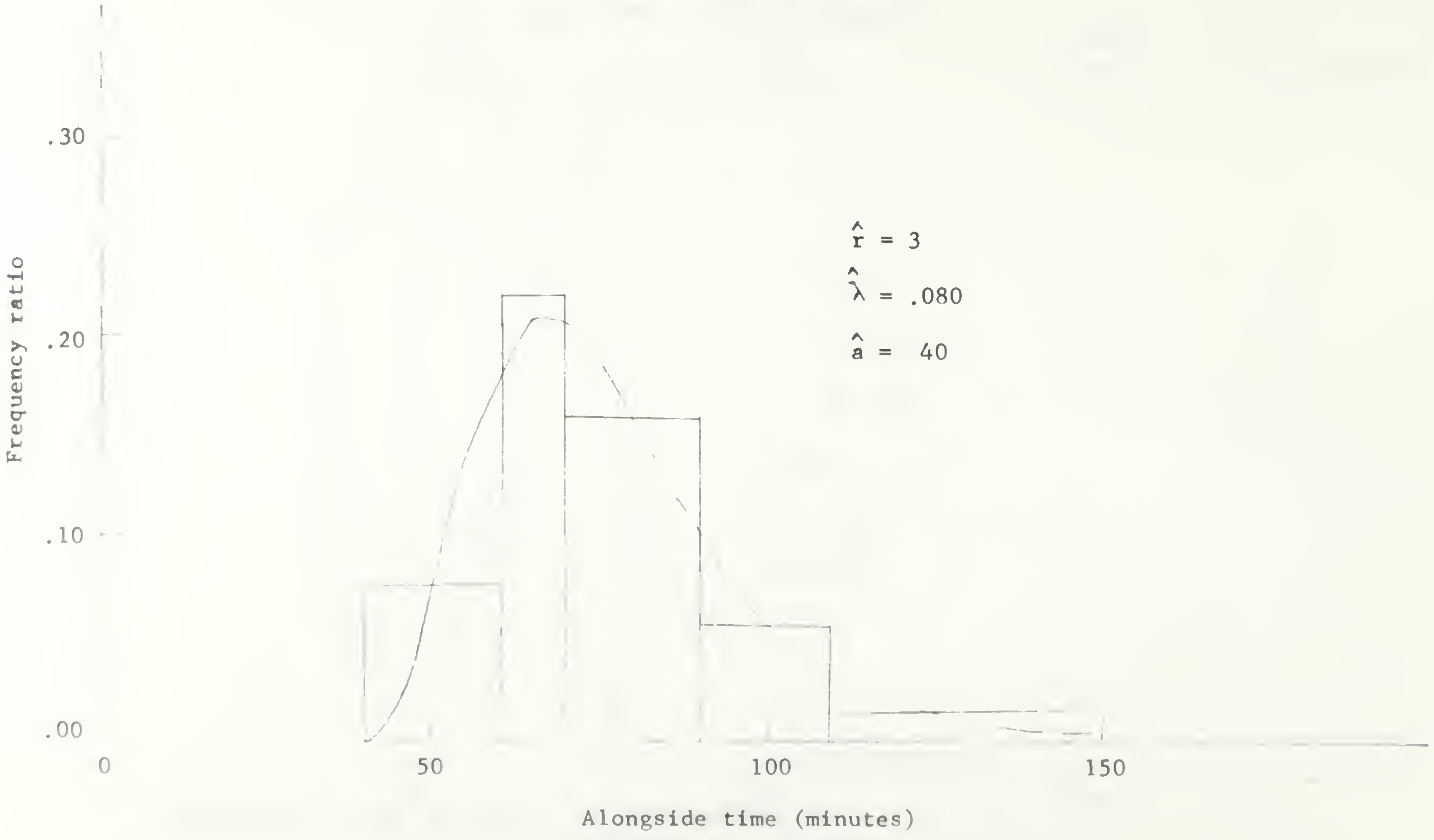


Figure 48. AE vs. DD - day. 0 - 1000 short tons

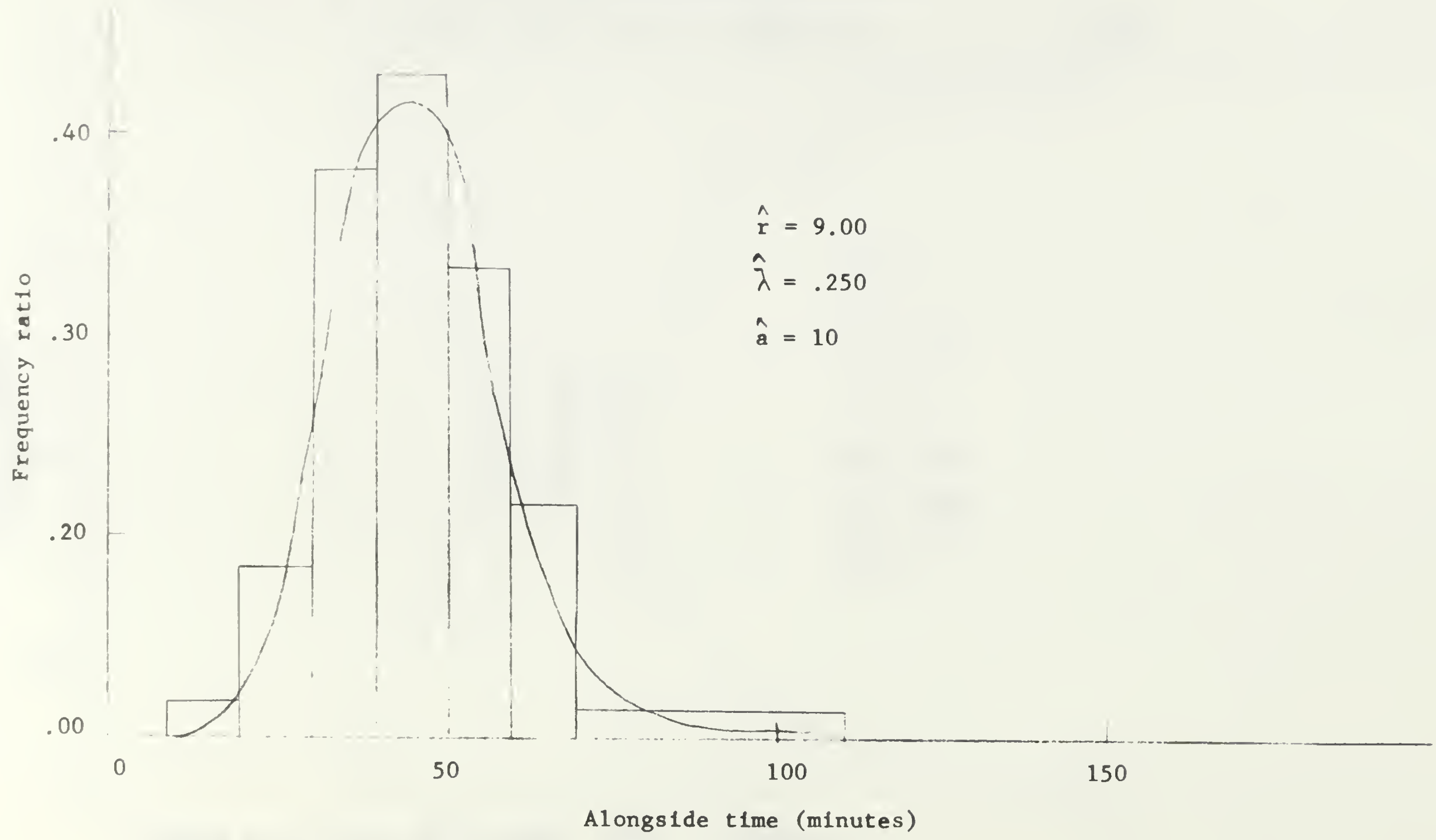


Figure 49. AE vs. DD - day. 1000 - 1999 short tons

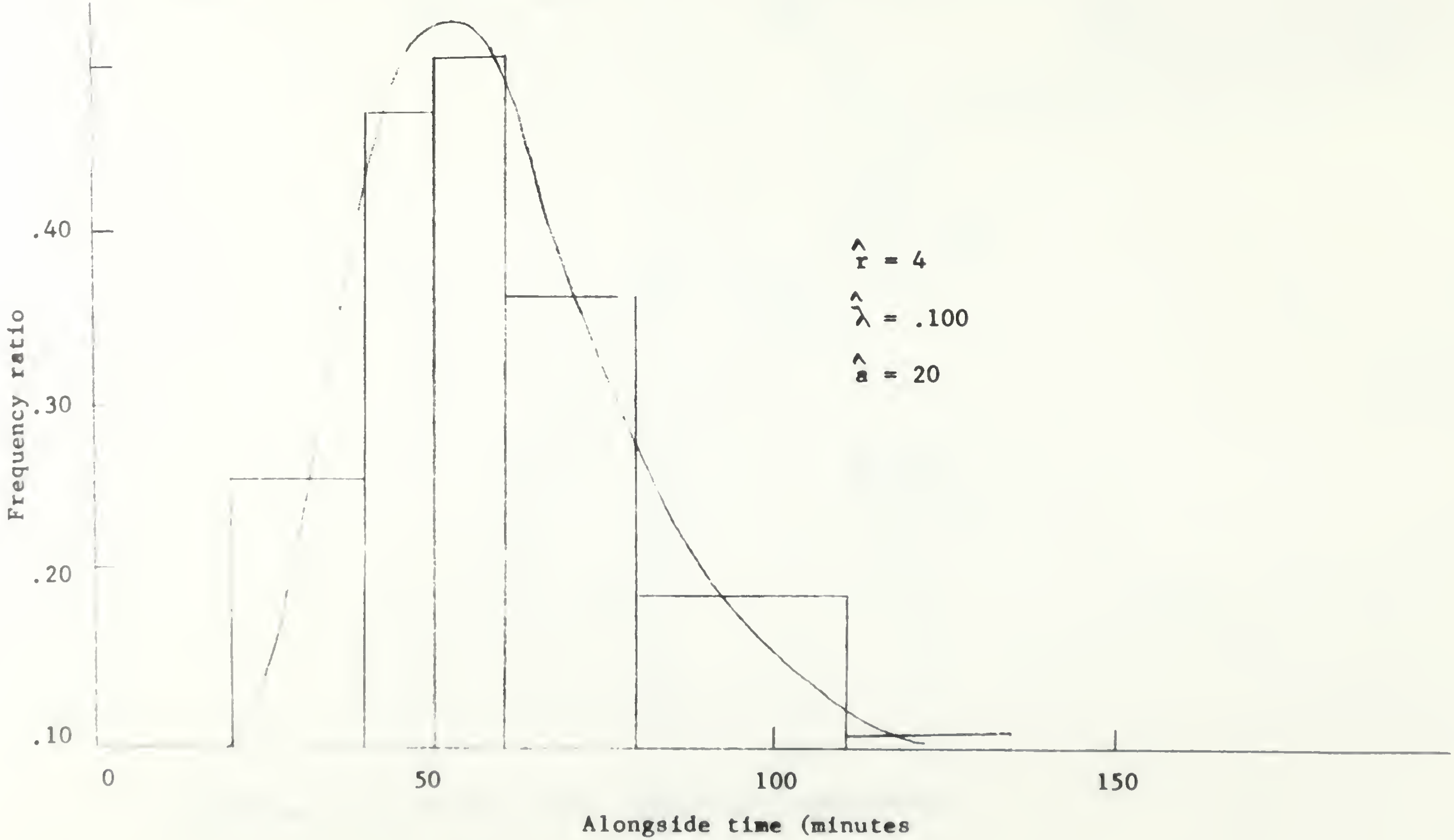


Figure 50. AE - vs. DD - day. 2000 - 2999 short tons

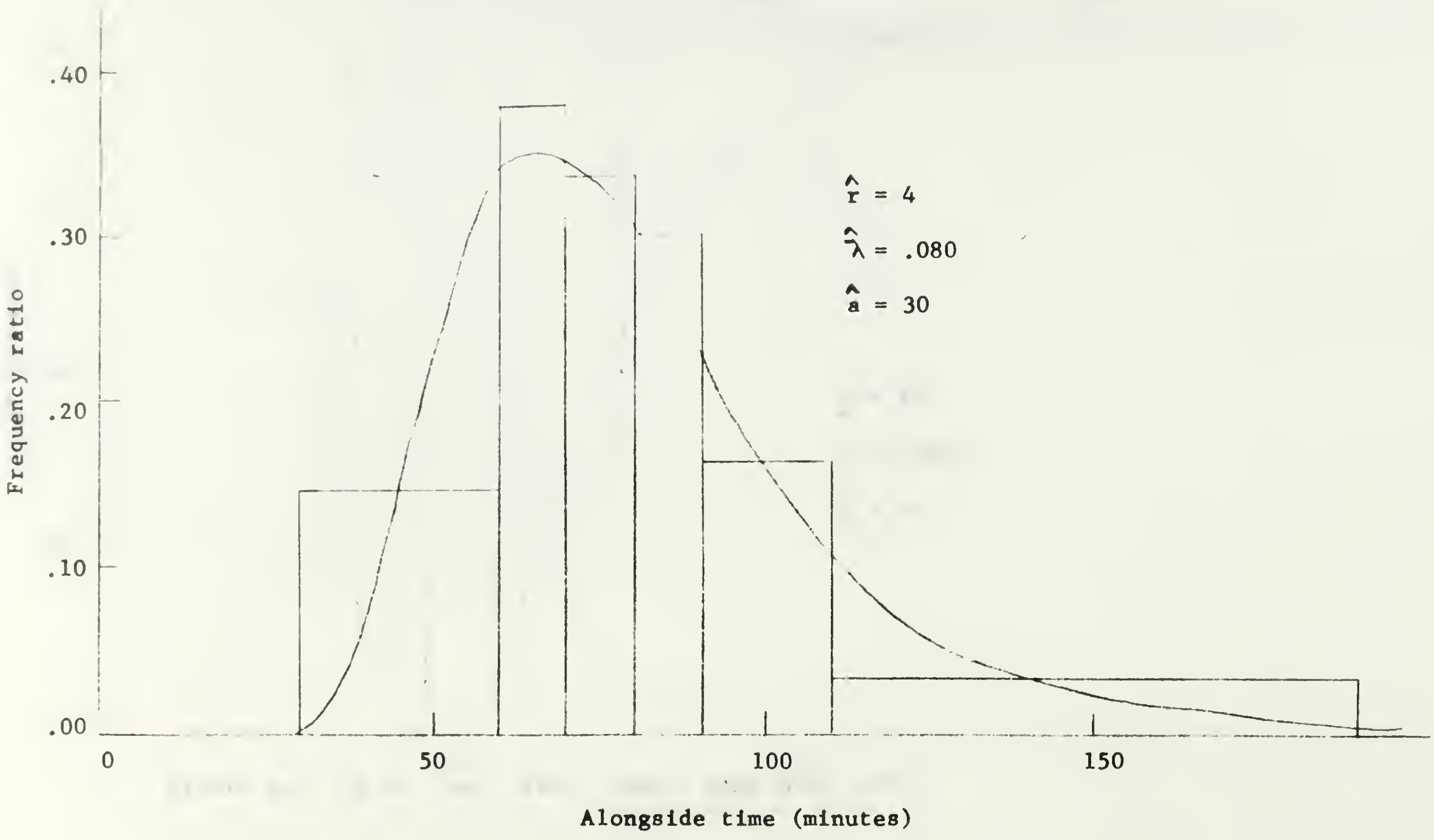


Figure 51. AE vs. DD - day. 3000 - 3999 short tons

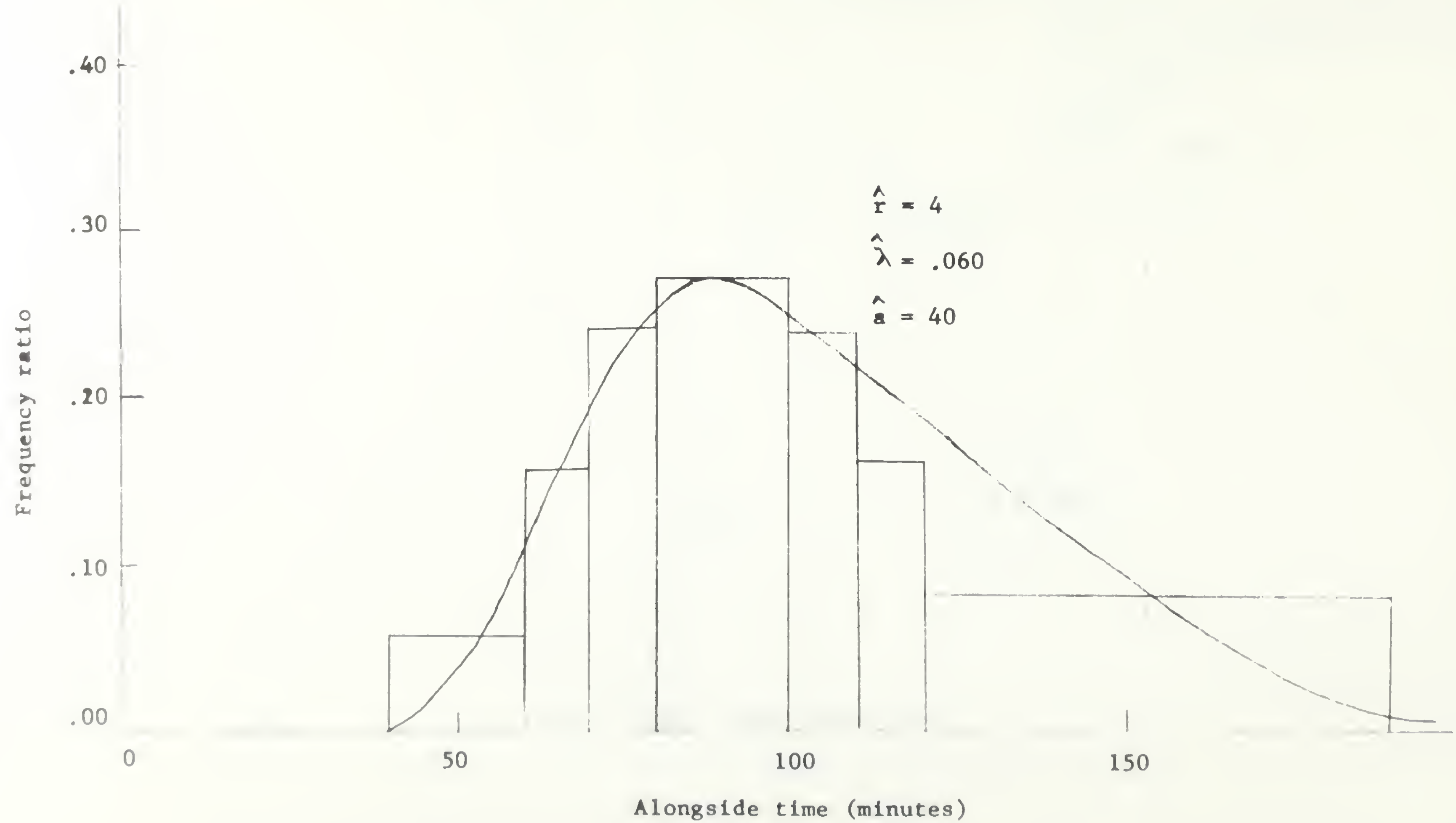


Figure 52. AE vs. DD - day. 4000 - 4999 short tons

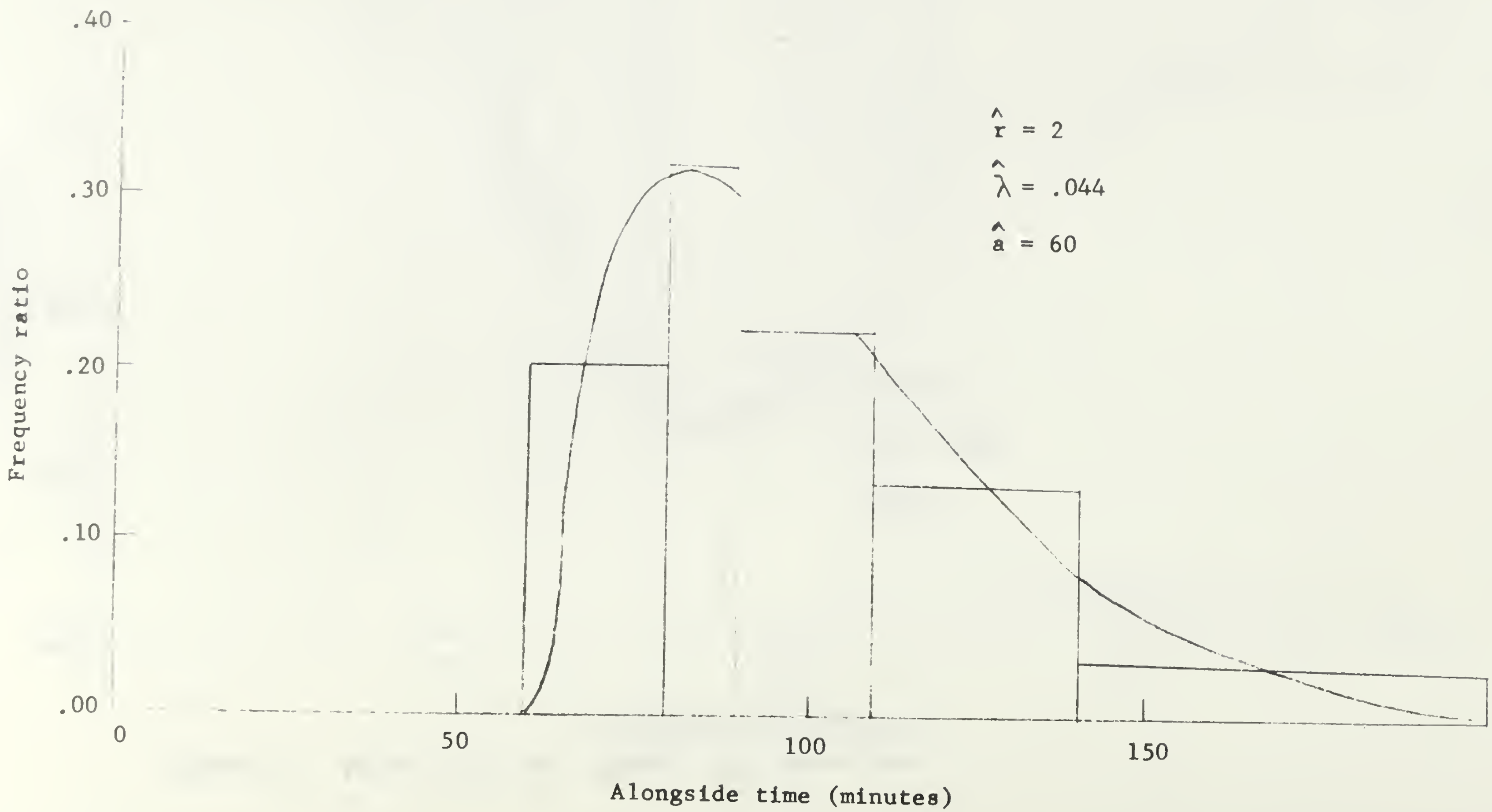


Figure 53. AE vs. DD - day. 5000 - short tons

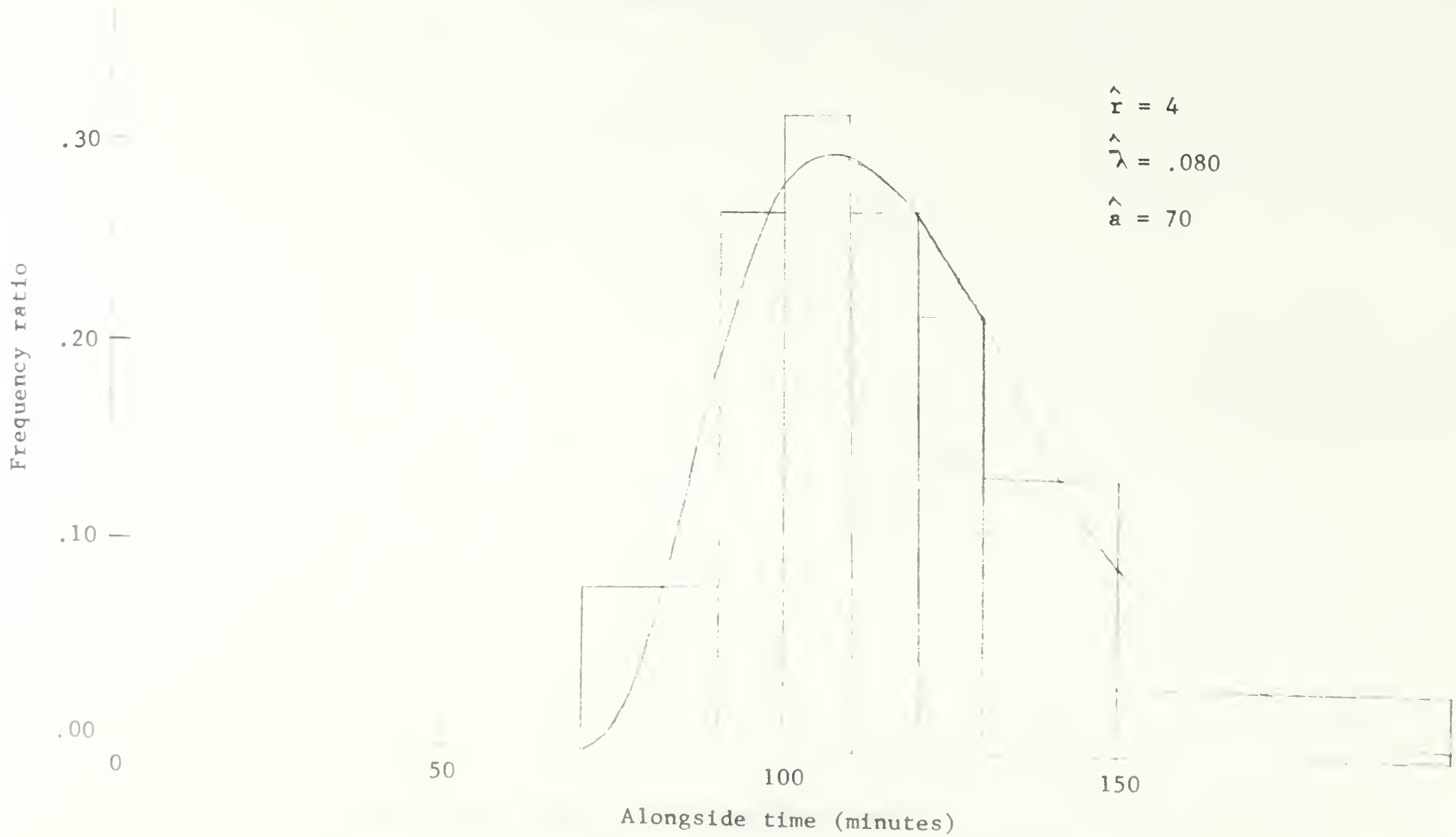


Figure 54. AE vs. DD - night. 0 - 1000 short tons

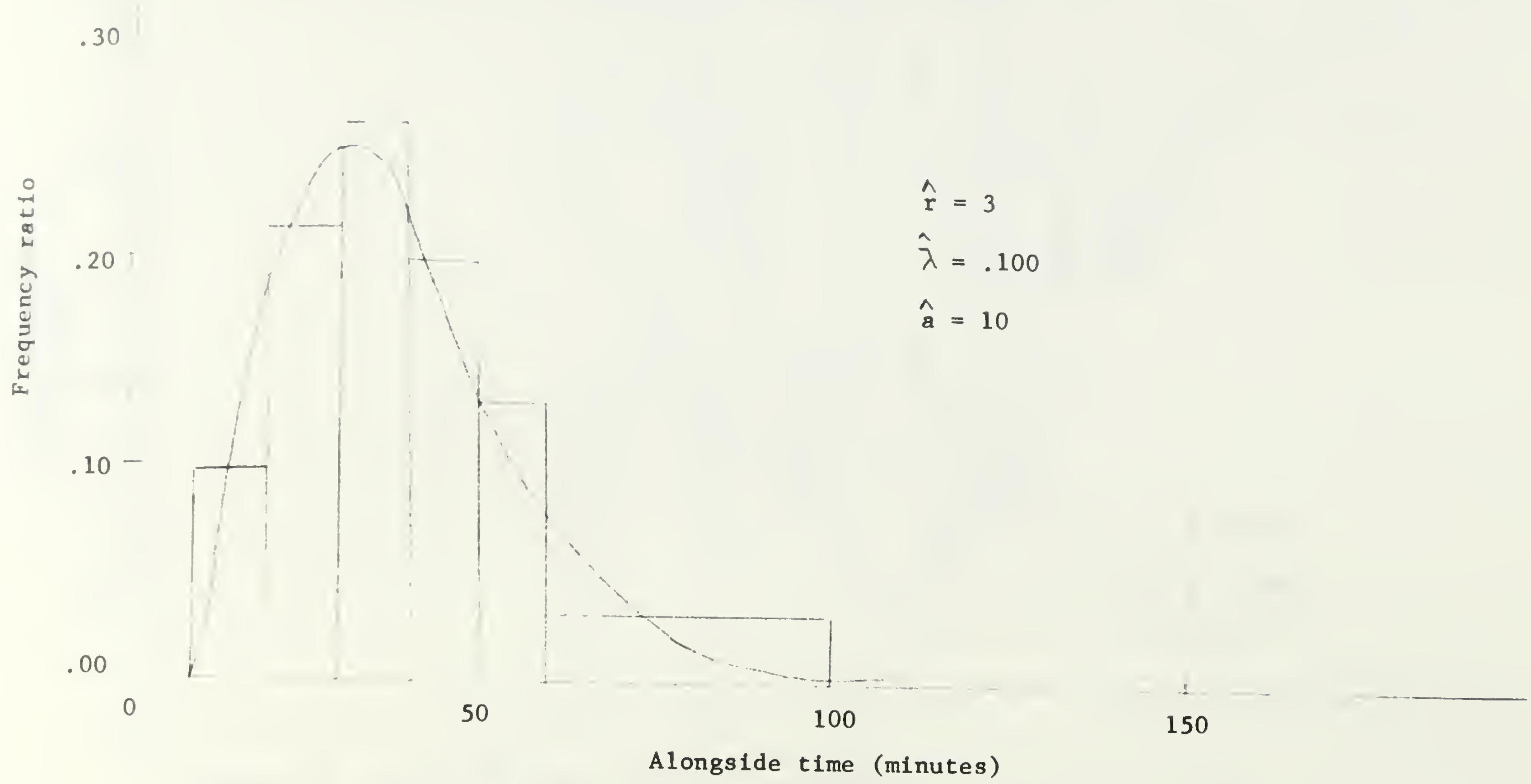


Figure 55. AE vs. DD - night. 1000 - 1999 short tons

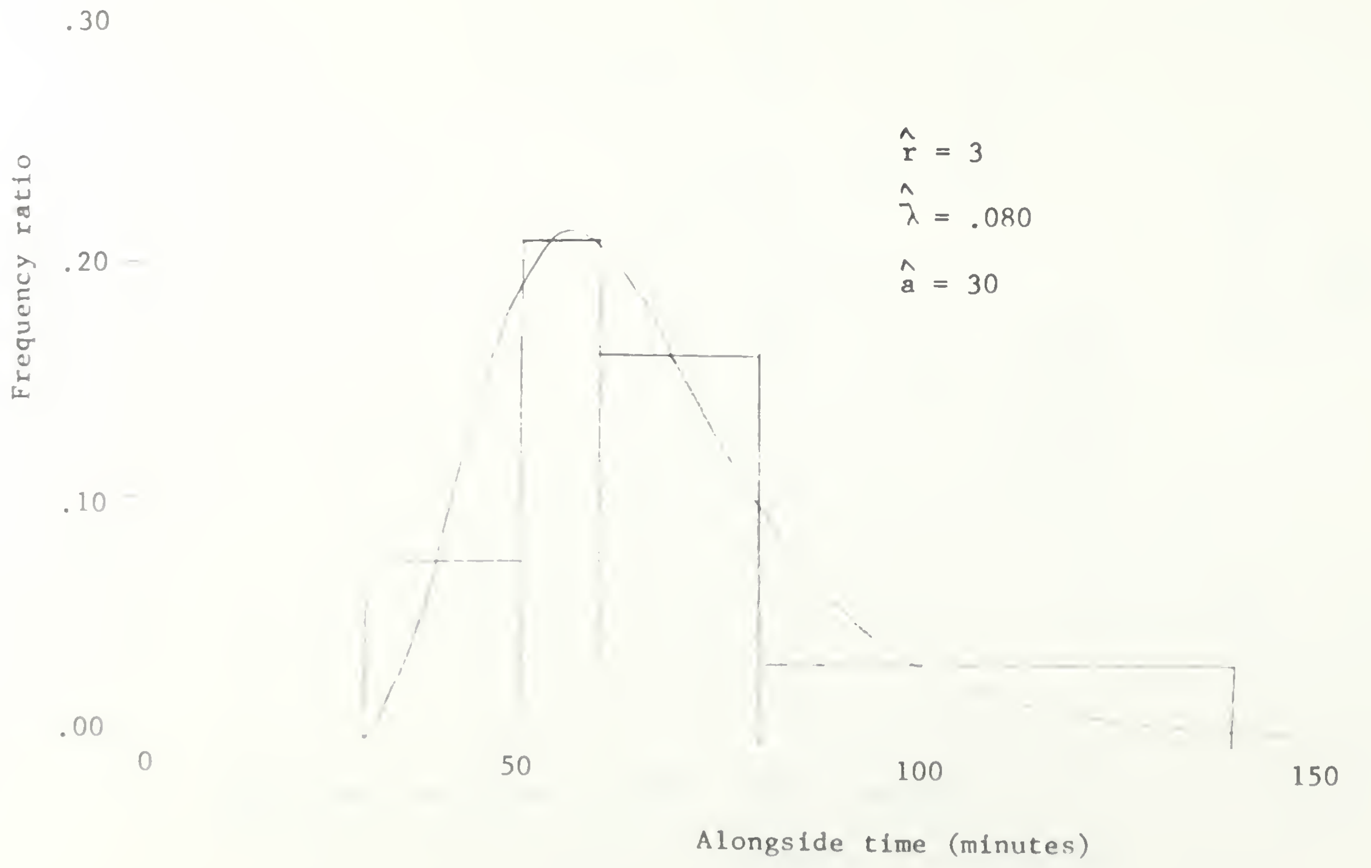


Figure 56. AE vs. DD - night. 2000 - 2999 short tons

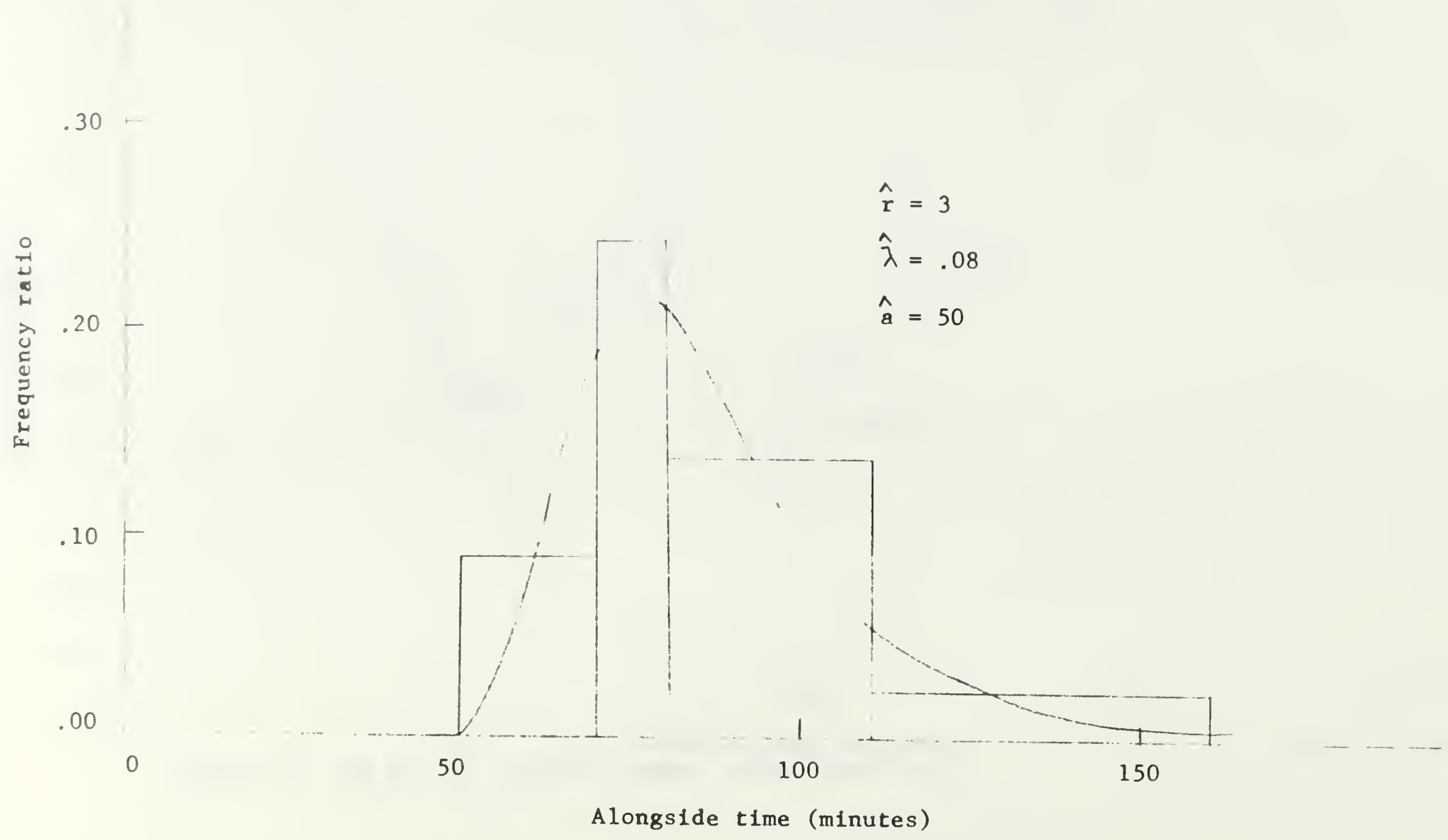


Figure 57. AE vs. DD - night. 3000 - 3999 short tons

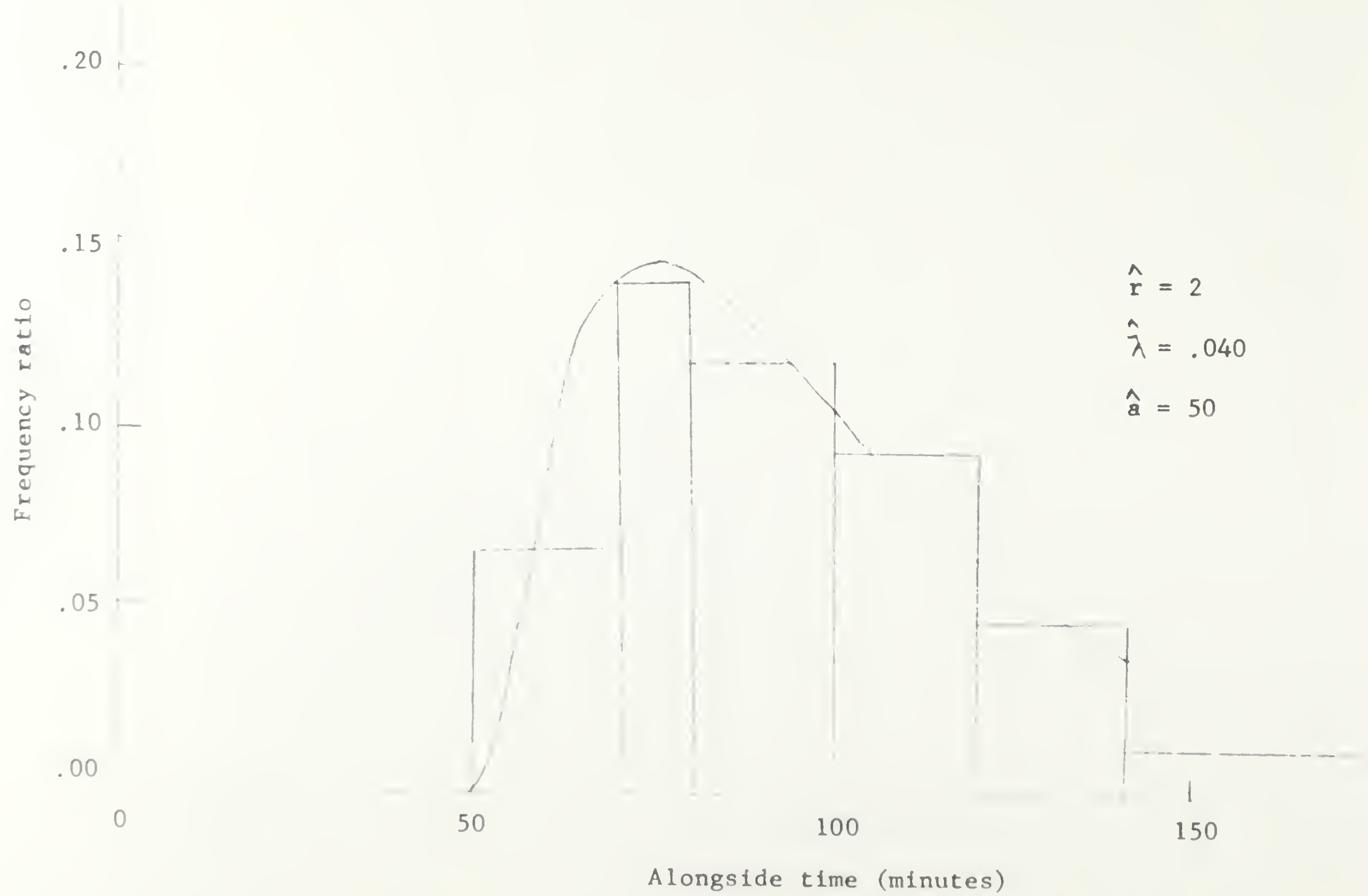
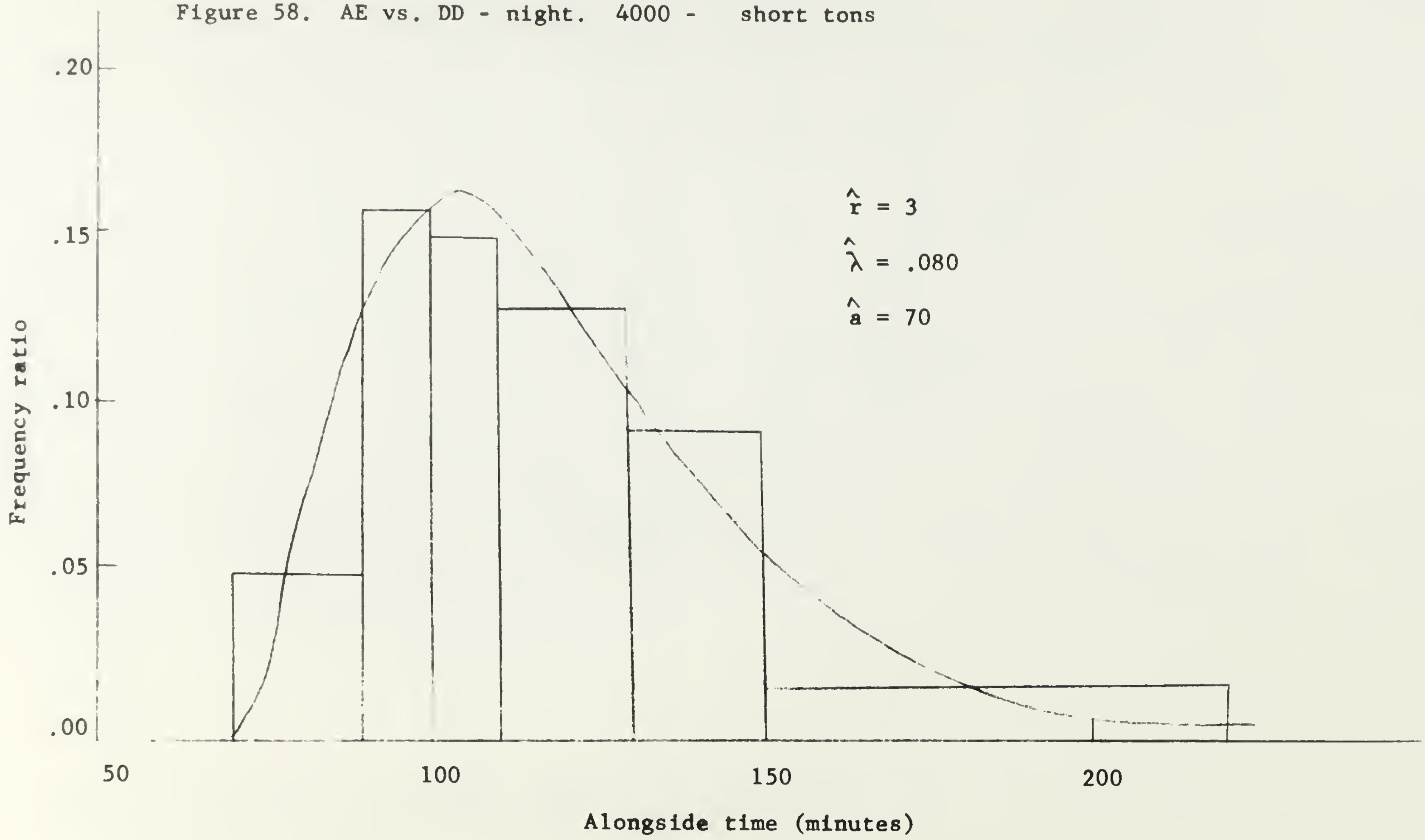


Figure 58. AE vs. DD - night. 4000 - short tons



APPENDIX D

Generating Random Observations from an Erland Distribution

The most important operation in a Monte Carlo type computer simulation is the generation of appropriate random variates. In this study, it was necessary to obtain random variates from the Erland distribution. Methematically the Erland distribution is a convolution of r independent identical exponential distributions, i.e., the distribution of the sum of r exponentially distributed, independent random variables whose density is given by

$$f(y) = e^{-\lambda y}, \ Y ? 0.$$

Thus, we are able to generate Erlang random variables by summing r independent exponentially distributed variables $Y_1, Y_2, Y_3, \ldots, Y_r$, each with expected value 1/ . Doing so we obtain that

$$x = \sum_{i=1}^{r} Y_i$$

has the Erland distribution with parameters r and , i.e. with density

$$f(x) = \begin{cases} \dfrac{\lambda^r x^{r-1} e^{-\lambda x}}{(r-1)!} & \text{for } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

The question now is how to obtain exponentially distributed random variates? The cumulative distribution function for the exponential distribution is given by

$$f(y) = 1-e^{-\lambda y}, \; y \geq 0.$$

It is well known that F(Y), and by symmetry 1-F(Y) also, is a uniform random variable over the interval (0,1) regardless of what the distribution of the continuous random variable Y is. This is equivalent to setting

$$W = e^{-\lambda Y}$$

where W is a random variable from the uniform (0,1) distribution. Therefore, taking the natural logarithm of both sides

$$-\lambda Y = \ln(W)$$

which yields

$$Y = -\frac{1}{\lambda} \ln(W).$$

Thus, if W is a uniform random variable over (0,1), Y will have the desired exponential distribution. Substituting into Eq. (1) we get

$$X = -\frac{1}{\lambda} \ln(\prod_{i=1}^{r} W_i).$$

There are several methods available for generating uniform (0,1) random variables. The IBM Scientific Subrouting RANDU was used in the study.

APPENDIX E

Computer Program

This appendix contains a printout of the computer program SIMA.

```
C        SIMA - SIMULATION OF UNDERWAY REPLENISHMENT OPERATION.          SIM00010
       DIMENSION ID(7),NQ(3),NS(7),NAC(7,12,3),NC(7,12,3),NIQ(7,3),XQ(7,3SIM00030
      1),NSD(6,4),MTEST(7,3),DXQ(7,3),LY(2,2,2),LMIN(2,2,2),LMAX(2,2,2),JSIM00040
      *QQ(20,2,2,2),AQQ(20,2,2,2),LCZ1(20,20,20),LCZ2(20,20,20),LCZ3(20,2SIM00050
      *0,20),NT(6,12,3),NTT(6),LTA(20,2,2,2),LCZ4(20,20,20)               SIM00060
       DATA LY/100,1000,1000,10000,100,1000,2000,10000/,LMIN/400,1000,400SIM00070
      *0,9000,600,1000,6000,20000/,LMAX/2200,5000,10000,40000,1800,4000,1SIM00080
      *2000,50000/                                                        SIM00090
       DATA LTA/2*20,30,20,30,20,30,30,20,7*30,4*40,10,20,30,40,60,70,14*SIM00100
      *0,20*0,20*0,7*30,2*40,30,4*40,6*0,10,30,50,50,70,15*0,40*0/       SIM00110
       DATA JQQ/2,3,2,2,3,2,4*3,5,1,2,5,4*3,1,3,9,3*4,2,4,14*0,8,24,14,9,SIM00120
      *7,22,6,4,12*0,7,12,18,27,18,15*0,7*3,2,2,3,4,3*3,6*0,3,3,3,2,3,15*SIM00130
      *0,8,12,9,19,11,15*0,8,6,13,24,24,15*0/                             SIM00140
       DATA AQQ/2*.1,.098,.1,.14,.06,2*.12,2*.1,.14,.04,.07,.14,4*.1,.04,SIM00150
      *.1,.25,.1,.08,.06,.044,.08,14*.0,.096,.244,.117,.074,.050,.156,.04SIM00160
      *7,.06,12*.0,.092,.114,.111,.141,.079,15*.0,10*.08,.16,3*.08,6*.0,.SIM00170
      *1,2*.08,.04,.08,15*.0,.068,.088,.063,.136,.056,15*.0,.084,.033,.07SIM00180
      *6,.113,.094,15*.0/,NSD/24*0/                                       SIM00190
C                                                                         SIM00200
C        INITIALIZE THE REPLENISHMENT SYSTEM                              SIM00210
C                                                                         SIM00220
       ITEST=0                                                            SIM00230
       IX=57835671                                                        SIM00240
       OMIN=1000.                                                         SIM00250
       T=0.                                                               SIM00260
       I=0                                                                SIM00270
       IP=0                                                               SIM00280
       DO 1 JE=1,6                                                        SIM00290
     1 NTT(JE)=0                                                          SIM00300
C                                                                         SIM00310
C        READ FROM DATA CARDS THE NUMBER AND TYPE OF SERVERS.             SIM00320
C                                                                         SIM00330
     2 I=I+1                                                              SIM00340
       READ(5,3) ID(I),NQ(I),NS(I),ICHE1                                  SIM00350
       IF(ICHE1)2,2,4                                                     SIM00360
     3 FORMAT(A3,2I2,I4)                                                  SIM00370
C                                                                         SIM00380
C        READ FROM DATA CARD THE DAY OR NIGHT DESIGNATION.                SIM00390
C                                                                         SIM00400
     4 READ (5,5) N                                                       SIM00410
     5 FORMAT (I2)                                                        SIM00420
C                                                                         SIM00430
C      READ FROM DATA CARDS THE INITIAL SITUATION AND THE LOAD REQUIRE-   SIM00440
C      MENTS. ML DESIGNATES THE SERVER IDENIFICATION NUMBER,KK THE NUMBERSIM00450
C      OF THE QUEUE FOR THE L TH SERVER, AND LL THE NUMBER IN THAT QUEUE.SIM00460
C                                                                         SIM00470
       DO 15 L=1,I                                                        SIM00480
       ML=NS(L)                                                           SIM00490
```

```
      MM=NQ(L)                                                          SIM00500
      KK=0                                                              SIM00510
    6 KK=KK+1                                                           SIM00520
      DO 7 JB=1,6                                                       SIM00530
    7 NT(JB,L,KK)=0                                                     SIM00540
      LL=0                                                              SIM00550
    8 LL=LL+1                                                           SIM00560
      READ(5,9) NAC(ML,LL,KK),NC(ML,LL,KK),LCZ1(ML,LL,KK),LCZ2(ML,LL,KK)SIM00570
     *,LCZ3(ML,LL,KK),LCZ4(ML,LL,KK),ICHE2                              SIM00580
    9 FORMAT (A3,I2,4I6,I1)                                             SIM00590
      DO 11 JA=1,6                                                      SIM00600
      IF(NC(ML,LL,KK).EQ.JA) GO TO 10                                   SIM00610
      GO TO 11                                                          SIM00620
   10 NT(JA,L,KK)=NT(JA,L,KK)+1                                         SIM00630
      NTT(JA)=NTT(JA)+1                                                 SIM00640
   11 CONTINUE                                                          SIM00650
      IF(ICHE2)8,8,12                                                   SIM00660
C                                                                       SIM00670
C       IP=THE TOTAL NUMBER OF COMBATANT SHIPS.                         SIM00680
C                                                                       SIM00690
   12 IP=IP+LL                                                          SIM00700
      DO 13 IJ=1,LL                                                     SIM00710
   13 WRITE (6,14) NAC(ML,IJ,KK),LCZ1(ML,IJ,KK),LCZ2(ML,IJ,KK),LCZ3(ML,ISIM00720
     *J,KK),LCZ4(ML,IJ,KK)                                              SIM00730
   14 FORMAT (5X,A3,4I9)                                                SIM00740
      NIQ (L,KK)=LL                                                     SIM00750
      IF(MM.EQ.2.AND.KK.EQ.1) GO TO 6                                   SIM00760
      IF(MM.LT.2) NIQ(L,KK+1)=-1                                        SIM00770
   15 CONTINUE                                                          SIM00780
      WRITE(6,16)                                                       SIM00790
   16 FORMAT (//,' ')                                                   SIM00800
      DO 19 J=1,I                                                       SIM00810
      JK=0                                                              SIM00820
   17 JK=JK+1                                                           SIM00830
      WRITE(6,18) ID(J),JK                                              SIM00840
   18 FORMAT (8X,A3,'/',I1)                                             SIM00850
      IF(NQ(J).EQ.2.AND.JK.EQ.1) GO TO 17                               SIM00860
   19 CONTINUE                                                          SIM00870
      WRITE (6,20)                                                      SIM00880
   20 FORMAT (8X,'TIME',//)                                             SIM00890
C                                                                       SIM00900
C       GENTRATE ALONGSIDE TIMES FOR ALL INITIAL CUSTOMERS              SIM00910
C                                                                       SIM00920
      DO 27 NN=1,I                                                      SIM00930
      MP=NS(NN)                                                         SIM00940
      LN=0                                                              SIM00950
   21 LN=LN+1                                                           SIM00960
      MTEST(NN,LN)=0                                                    SIM00970
```

```
C                                                                          SIM00980
C      DETERMINE THE LOAD REQUIREMENT                                      SIM00990
C                                                                          SIM01000
      MR=NC(MP,1,LN)                                                       SIM01010
      IF(MP.EQ.1) LOAD=LCZ1(MP,1,LN)                                       SIM01020
      IF(MP.EQ.2) LOAD=LCZ2(MP,1,LN)                                       SIM01030
      IF(MP.EQ.3) LOAD=LCZ3(MP,1,LN)                                       SIM01040
      IF(MP.EQ.4) LOAD=LCZ4(MP,1,LN)                                       SIM01050
C                                                                          SIM01060
C      DETERMINE LOAD INTERVAL                                             SIM01070
C                                                                          SIM01080
      IF(LOAD.LT.LMIN(MP,MR,N)) GO TO  24                                  SIM01090
      K=0                                                                  SIM01100
   22 K=K+1                                                                SIM01110
      TEMP=(K*LY(MP,MR,N))+LMIN(MP,MR,N)                                   SIM01120
      IF(TEMP.GT.LMAX(MP,MR,N)) GO TO 23                                   SIM01130
      IF(LOAD.LE.TEMP) GO TO 25                                            SIM01140
      GO TO 22                                                             SIM01150
   23 L=K+1                                                                SIM01160
      GO TO 26                                                             SIM01170
   24 L=1                                                                  SIM01180
      GO TO 26                                                             SIM01190
   25 L=K+1                                                                SIM01200
   26 AAA=AQQ(L,MP,MR,N)                                                   SIM01210
      NAA=JQQ(L,MP,MR,N)                                                   SIM01220
C                                                                          SIM01230
C      GENERATE ALONGSIDE TIMES FOR THE SHIPS INITIALLY BEING SERVED.      SIM01240
C                                                                          SIM01250
      CALL GAMMA(IX,NAA,AAA,XB,IX)                                         SIM01260
      XB=XB+LTA(L,MP,MR,N)                                                 SIM01270
      XQ(NN,LN)=XB                                                         SIM01280
      IF(NQ(NN).EQ.2.AND.LN.EQ.1) GO TO 21                                 SIM01290
   27 CONTINUE                                                             SIM01300
C                                                                          SIM01310
C      PRINT OUT INITIAL CONDITION VECTOR AND TIME.                        SIM01320
C                                                                          SIM01330
      DO 30 JI=1,I                                                         SIM01340
      MJ=0                                                                 SIM01350
   28 MJ=MJ+1                                                              SIM01360
      WRITE(6,29) NIQ(JI,MJ)                                               SIM01370
   29 FORMAT (8X,I3)                                                       SIM01380
      IF(NQ(JI).EQ.2.AND.MJ.EQ.1) GO TO  28                                SIM01390
   30 CONTINUE                                                             SIM01400
      WRITE (6,31) T                                                       SIM01410
   31 FORMAT (8X,F7.2,//)                                                  SIM01420
C                                                                          SIM01430
C       AT THIS POINT ALL INITIAL CONDITIONS ARE SET AND THE PROGRAM       SIM01440
C       ENTERS A LOOP INCREMENTING THE REPLENISHMENT SYSTEM AS             SIM01450
```

```
C        APPROPRIATE.                                                    SIM01460
C                                                                        SIM01470
   32 DO 41 IH=1,I                                                       SIM01480
      IS=0                                                               SIM01490
   33 IS=IS+1                                                            SIM01500
      IF(MTEST(IH,IS).EQ.1.AND.NIQ(IH,IS).NE.0) GO TO 34                 SIM01510
      GO TO 40                                                           SIM01520
   34 IT=NS(IH)                                                          SIM01530
      IR=NC(IH,1,IS)                                                     SIM01540
C                                                                        SIM01550
C       DETERMINE LOAD REQUIREMENT                                       SIM01560
C                                                                        SIM01570
      IF(IT.EQ.1) LOAD = LCZ1(IH,1,IS)                                   SIM01580
      IF(IT.EQ.2) LOAD=LCZ2(IH,1,IS)                                     SIM01590
      IF(IT.EQ.3) LOAD=LCZ3(IH,1,IS)                                     SIM01600
      IF(IT.EQ.4) LOAD=LCZ4(IH,1,IS)                                     SIM01610
C                                                                        SIM01620
C        DETERMINE LOAD INTERVAL                                         SIM01630
C                                                                        SIM01640
      IF(LOAD.LT.LMIN(IT,IR,N)) GO TO 37                                 SIM01650
      K=0                                                                SIM01660
   35 K=K+1                                                              SIM01670
      TEMP=(K*LY(IT,IR,N))+LMIN(IT,IR,N)                                 SIM01680
      IF(TEMP.GT.LMAX(IT,IR,N)) GO TO 36                                 SIM01690
      IF(LOAD.LE.TEMP) GO TO 38                                          SIM01700
      GO TO 35                                                           SIM01710
   36 L=K+1                                                              SIM01720
      GO TO 39                                                           SIM01730
   37 L=1                                                                SIM01740
      GO TO 39                                                           SIM01750
   38 L=K+1                                                              SIM01760
   39 AUU=AQQ(L,IT,IR,N)                                                 SIM01770
      IUU=JQQ(L,IT,IR,N)                                                 SIM01780
C                                                                        SIM01790
C       GENERATE ALONGSIDE TIME                                          SIM01800
C                                                                        SIM01810
      CALL GAMMA (IX,IUU,AUU,YYC,IX)                                     SIM01820
      YY=YYC+LTA(L,IT,IR,N)                                              SIM01830
      XQ(IH,IS)=T+YY                                                     SIM01840
      MTEST(IH,IS)=0                                                     SIM01850
   40 IF(NQ(IH).EQ.2.AND.IS.EQ.1) GO TO 33                               SIM01860
   41 CONTINUE                                                           SIM01870
C                                                                        SIM01880
C       INCREMENT THE SYSTEM AND PRINT OUT CONDITION VECTOR.             SIM01890
C                                                                        SIM01900
      DO 45 MI=1,I                                                       SIM01910
      MA=0                                                               SIM01920
   42 MA=MA+1                                                            SIM01930
```

```
      IF(XQ(MI,MA)-OMIN) 43,44,44                                       SIM01940
   43 OMIN=XQ(MI,MA)                                                    SIM01950
      MB=MI                                                             SIM01960
      MC=MA                                                             SIM01970
   44 IF(NQ(MI).EQ.2.AND.MA.EQ.1) GO TO 42                              SIM01980
   45 CONTINUE                                                          SIM01990
      T=OMIN                                                            SIM02000
      MS=NS(MB)                                                         SIM02010
      JD=NC(MS,1,MC)                                                    SIM02020
      IF(NIQ(MB,MC).GE.1) GO TO 46                                      SIM02030
      GO TO 47                                                          SIM02040
   46 NIQ(MB,MC)=NIQ(MB,MC)-1                                           SIM02050
      IF(NIQ(MB,MC).NE.0) MTEST(MB,MC)=1                                SIM02060
      NT(JD,MB,MC)=NT(JD,MB,MC)-1                                       SIM02070
   47 NSD(JD,MB)=NSD(JD,MB)+1                                           SIM02080
C                                                                       SIM02090
C     NSD(JC,MB)= NO. OF JD TYPE SHIPS SERVED BY SERVER MB              SIM02100
C                                                                       SIM02110
      IF(MB.EQ.I) GO TO 50                                              SIM02120
      MT=NS(MB+1)                                                       SIM02130
      MU=NIQ(MB+1,1)                                                    SIM02140
      MV=NIQ(MB+1,2)                                                    SIM02150
      IF(NIQ(MB+1,2).EQ.-1) GO TO 48                                    SIM02160
      NZ=NSD(JD,MB+1)+NT(JD,MB+1,1)+NT(JD,MB+1,2)                       SIM02170
      IF(NZ.EQ.NTT(JD)) GO TO 53                                        SIM02180
      IF(NIQ(MB+1,1).LE.NIQ(MB+1,2))GO TO 49                            SIM02190
      IF(NIQ(MB+1,2).EQ.0) MTEST(MB+1,2)=1                              SIM02200
      NIQ(MB+1,2)=NIQ(MB+1,2)+1                                         SIM02210
      NT(JD,MB+1,2)=NT(JD,MB+1,2)+1                                     SIM02220
      NC(MT,MV+1,2)=NC(MS,1,MC)                                         SIM02230
      LCZ1(MT,MV+1,2)=LCZ1(MS,1,MC)                                     SIM02240
      LCZ2(MT,MV+1,2)=LCZ2(MS,1,MC)                                     SIM02250
      LCZ3(MT,MV+1,2)=LCZ3(MS,1,MC)                                     SIM02260
      LCZ4(MT,MV+1,2)=LCZ4(MS,1,MC)                                     SIM02270
      GO TO 53                                                          SIM02280
   48 NY=NSD(JD,MB+1)+NT(JD,MB+1,1)                                     SIM02290
      IF(NY.EQ.NTT(JD)) GO TO 53                                        SIM02300
   49 IF(NIQ(MB+1,1).EQ.0) MTEST(MB+1,1)=1                              SIM02310
      NIQ(MB+1,1)=NIQ(MB+1,1)+1                                         SIM02320
      NT(JD,MB+1,1)=NT(JD,MB+1,1)+1                                     SIM02330
      NC(MT,MU+1,1)=NC(MS,1,MC)                                         SIM02340
      LCZ1(MT,MU+1,1)=LCZ1(MS,1,MC)                                     SIM02350
      LCZ2(MT,MU+1,1)=LCZ2(MS,1,MC)                                     SIM02360
      LCZ3(MT,MU+1,1)=LCZ3(MS,1,MC)                                     SIM02370
      LCZ4(MT,MU+1,1)=LCZ4(MS,1,MC)                                     SIM02380
      GO TO 53                                                          SIM02390
   50 MW=NS(1)                                                          SIM02400
      MX=NIQ(1,1)                                                       SIM02410
```

```
      MY=NIQ(1,2)                                              SIM02420
      IF(NIQ(1,2).EQ.-1) GO TO 51                              SIM02430
      NX=NSD(JD,1)+NT(JD,1,1)+NT(JD,1,2)                       SIM02440
      IF(NX.EQ.NTT(JD)) GO TO 53                               SIM02450
      IF(NIQ(1,1).LE.NIQ(1,2))GO TO 52                         SIM02460
      IF(NIQ(1,2).EQ.0) MTEST(1,2)=1                           SIM02470
      NIQ(1,2)=NIQ(1,2)+1                                      SIM02480
      NT(JD,1,2)=NT(JD,1,2)+1                                  SIM02490
      NC(MW,MY+1,2)=NC(MS,1,MC)                                SIM02500
      LCZ1(MW,MY+1,2)=LCZ1(MS,1,MC)                            SIM02510
      LCZ2(MW,MY+1,2)=LCZ2(MS,1,MC)                            SIM02520
      LCZ3(MW,MY+1,2)=LCZ3(MS,1,MC)                            SIM02530
      LCZ4(MW,MY+1,2)=LCZ3(MS,1,MC)                            SIM02540
      GO TO 53                                                 SIM02550
   51 NW=NSD(JD,1)+NT(JD,1,1)                                  SIM02560
      IF(NW.EQ.NTT(JD)) GO TO 53                               SIM02570
   52 IF(NIQ(1,1).EQ.0) MTEST(1,1)=1                           SIM02580
      NIQ(1,1)=NIQ(1,1)+1                                      SIM02590
      NT(JD,1,1)=NT(JD,1,1)+1                                  SIM02600
      NC(MW,MX+1,1)=NC(MS,1,MC)                                SIM02610
      LCZ1(MW,MX+1,1)=LCZ1(MS,1,MC)                            SIM02620
      LCZ2(MW,MX+1,1)=LCZ2(MS,1,MC)                            SIM02630
      LCZ3(MW,MX+1,1)=LCZ3(MS,1,MC)                            SIM02640
      LCZ4(MW,MX+1,1)=LCZ4(MS,1,MC)                            SIM02650
   53 IF(NIQ(MB,MC).GT.0) GO TO 54                             SIM02660
      GO TO 57                                                 SIM02670
   54 IF(NIQ(MB,MC).GE.2) GO TO 55                             SIM02680
      IF(NIQ(MB,MC).NE.1) GO TO 57                             SIM02690
      NC(MS,1,MC)=NC(MS,2,MC)                                  SIM02700
      LCZ1(MS,1,MC)=LCZ1(MS,2,MC)                              SIM02710
      LCZ2(MS,1,MC)=LCZ2(MS,2,MC)                              SIM02720
      LCZ3(MS,1,MC)=LCZ3(MS,2,MC)                              SIM02730
      LCZ4(MS,1,MC)=LCZ4(MS,2,MC)                              SIM02740
      GO TO 57                                                 SIM02750
   55 MD=NIQ(MB,MC)                                            SIM02760
      DO 56 ME=1,MD                                            SIM02770
      NC(MS,ME,MC)=NC(MS,ME+1,MC)                              SIM02780
      LCZ1(MS,ME,MC)=LCZ1(MS,ME+1,MC)                          SIM02790
      LCZ2(MS,ME,MC)=LCZ2(MS,ME+1,MC)                          SIM02800
      LCZ3(MS,ME,MC)=LCZ3(MS,ME+1,MC)                          SIM02810
   56 LCZ4(MS,ME,MC)=LCZ4(MS,ME+1,MC)                          SIM02820
   57 DO 59 KA=1,I                                             SIM02830
      KB=0                                                     SIM02840
   58 KB=KB+1                                                  SIM02850
      WRITE(6,29) NIQ(KA,KB)                                   SIM02860
      IF(NQ(KA).EQ.2.AND.KB.EQ.1) GO TO 58                     SIM02870
   59 CONTINUE                                                 SIM02880
      WRITE (6,31) T                                           SIM02890
```

```
      IF(NIQ(MB,MC).EQ.0) ITEST=0                                       SIM02900
      XQ(MB,MC)=100000.                                                 SIM02910
      DO 61 MH=1,I                                                      SIM02920
      NU=0                                                              SIM02930
   60 NU=NU+1                                                           SIM02940
      IF(NIQ(MH,NU).NE.0) ITEST=1                                       SIM02950
      IF(NQ(MH).EQ.2.AND.NU.EQ.1) GO TO 60                              SIM02960
   61 CONTINUE                                                          SIM02970
      IF(ITEST.EQ.0) GO TO 62                                           SIM02980
      OMIN=XQ(MB,MC)                                                    SIM02990
      GO TO 32                                                          SIM03000
   62 STOP                                                              SIM03010
      END                                                               SIM03020
```

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13. ABSTRACT

By analyzing CONREP data for oiler (AO) and ammunition (AE) ships replenishing destroyers (DD) and attack carriers (CVA) with regard to day and night operation and load requirement, it is shown through Chi-Square goodness-of-fit tests that alongside time data can be fitted to an Erlang or exponential distribution. In addition, several methods for estimating the shape, scale, and shift parameters for gamma and Erlang distributions are presented.

Employing well known but analytically little used features of the replenishment operation, a computer simulation model is then formulated and programmed based on these distributions, and thus sensitive to changes in load requirements. Using the simulation as an experimental device, an example is run to demonstrate its use in estimating the time to complete the operation and conducting sensitivity analyses on the various load requirements.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
underway replenishment						
simulation						
Erlang distribution						
gamma distribution						